

$$f(x+r) = \begin{cases} rx+2 & x \geq 1 \\ rx-1 & x < 1 \end{cases}$$

$$y = \sqrt{f(r-x) - f(x+1)}$$

سوال ①

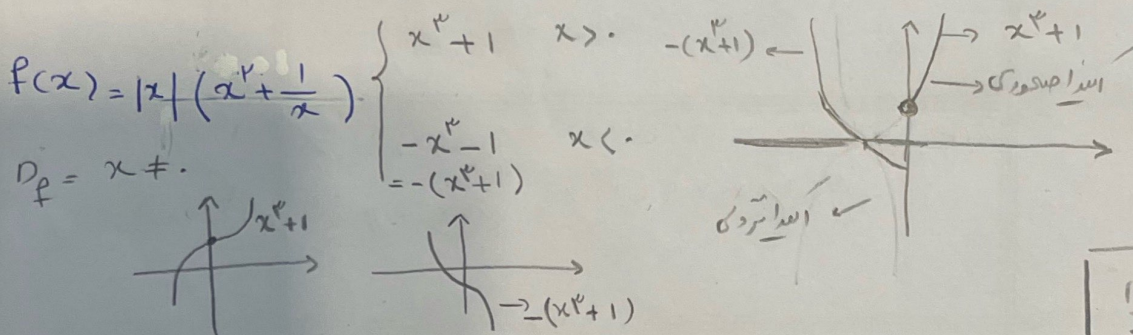
$$f(r-x) - f(x+1) \geq 0 \Rightarrow f(r-x) \geq f(x+1) \Rightarrow r-x \geq x+1 \Rightarrow \boxed{x \leq 1}$$

$$f(x) = \begin{cases} r(x+r)+2 & x+r \geq 1 \Rightarrow x \geq -1 \\ r(x+r)-1 & x+r < 1 \Rightarrow x < -1 \end{cases}$$

$$f(x) = (x^r + x)^r \quad f \circ f(x) < f(x^r)$$

سوال ②

$$f(f(x)) < f(x^r) \Rightarrow f(x) < x^r \Rightarrow x^r > (x^r + x)^r \Rightarrow x > x^r + x \Rightarrow x^r < 0 \Rightarrow \boxed{x < 0}$$



سوال ③

$$f(x) = \frac{rx+1}{|x|-|x+1|} \quad (-\infty, \alpha] \quad \alpha_{\max} = ?$$

سوال ④

$$\left. \begin{aligned} \frac{rx+1}{x-x-1} &= -(rx+1) & x > 0 \\ \frac{rx+1}{-x-x-1} &= \frac{rx+1}{-2x-1} & -1 \leq x \leq 0 \\ \frac{rx+1}{-x-(-x-1)} &= \frac{rx+1}{1} & x < -1 \end{aligned} \right\} \Rightarrow$$

$$\frac{rx+1}{|x|-|x+1|} \times \frac{|x|+|x+1|}{|x|+|x+1|} = \frac{(rx+1)(|x|+|x+1|)}{x^2 - (x+1)^2} = \frac{(rx+1)(|x|+|x+1|)}{-(rx+1)} = -(|x|+|x+1|)$$

$\Rightarrow \boxed{\alpha \leq 0}$

$$y = \log_{\frac{1}{r}} (x^2 - 2x - 1)$$

$$x^2 - 2x - 1 > 0 \quad (x-4)(x+2) > 0$$

سؤال ٥

$$\begin{array}{c|ccc} & -2 & & 4 \\ \hline & + & - & + \\ \hline & \infty & & \infty \end{array}$$

$$\begin{cases} x < -2 \\ x > 4 \end{cases}$$

$$x > 4 \rightarrow \log_{\frac{1}{r}} (x^2 - 2x - 1) \Rightarrow \text{صورتی است} \checkmark$$

$$y = \log_{\frac{1}{r}} (x^2 - 2x - 1) = -\log_r (x^2 - 2x - 1)$$

$$x < -2 \rightarrow x^2 - 2x - 1 \Rightarrow \text{تزیلی است } x$$

$$\Rightarrow \log_{\frac{1}{r}} (x^2 - 2x - 1) \Rightarrow \text{باید تزیلی باشد}$$

$$\boxed{x < -2}$$

$$f(x) = (a^2 - 3)^x$$

سؤال ٤

$$a^2 - 3 > 1 \Rightarrow a^2 > 4 \Rightarrow \boxed{a > 2 \cup a < -2}$$

الف) تابع f صورتی است.

$$a^2 - 3 \geq 1 \Rightarrow a^2 \geq 4 \Rightarrow \boxed{a \geq 2, a \leq -2}$$

ب) تابع f صورتی است.

$$\underline{\text{دانش}} \quad x_1 < x_2 \Rightarrow f(x_1) \leq f(x_2) \Rightarrow a^2 - 3 = 0 \Rightarrow a^2 = 3 \Rightarrow \boxed{a = \pm \sqrt{3}}$$

$$f(x) = x^3 + ax^2 + bx + c$$

$$(-3, -1)$$

$$y = |f(x)| \rightarrow (k, +\infty) \text{ است}$$

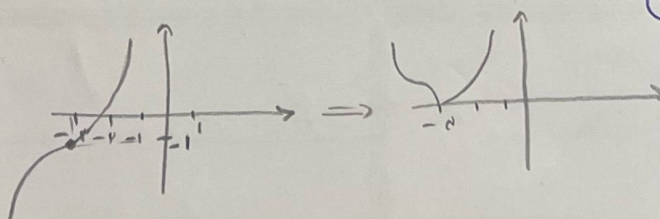
$$(x+3)^3 \rightarrow (x+3)^3 = 1$$

$$y = |f(x)| \rightarrow y = |(x+3)^3 - 1|$$

$$|(x+3)^3 - 1| = 0 \rightarrow (x+3)^3 - 1 = 0$$

$$(x+3)^3 = 1 \xrightarrow{\sqrt[3]{}} x+3 = 1 \rightarrow \boxed{x = -2}$$

$$\Rightarrow \boxed{k = -2}$$



سؤال ٧

$$f(x) = x + \sqrt{x} - 2$$

$$\sqrt{x} = t \Rightarrow t^2 + t - 2 \quad / \Delta > 0 \text{ اتي } \sqrt{x} > 0 \text{ ضروري}$$

سؤال ٨

$$f \circ f(x) \leq f(x)$$

$$f \circ f(x) \leq f(\sqrt{x}) \Rightarrow f(x) \leq \sqrt{x} \rightarrow x + \sqrt{x} - 2 \leq \sqrt{x} \Rightarrow x - 2 \leq 0 \Rightarrow x \leq 2$$

$$\sqrt{x} \Rightarrow x \geq 0 \quad \cdot \leq x \leq 2 \Rightarrow$$

$$\boxed{\left[-\frac{1}{2}, 2 \right]} \quad \boxed{0, 1, 2}$$

$$f(x) = \sqrt[3]{4 \cos^2 x - 1} - \sqrt[3]{1 - 4 \cos^2 x} \quad [a, b] \quad b - a = ?$$

سؤال ٩

$$-1 \leq \cos^2 x \leq 1 \rightarrow -1 \leq 4 \cos^2 x \leq 4 \Rightarrow -1 \leq 4 \cos^2 x - 1 \leq 3 \Rightarrow 1 \leq \sqrt[3]{4 \cos^2 x - 1} \leq 3$$

$$\Rightarrow \sqrt[3]{4 \cos^2 x - 1} = t$$

$$f(t) = \sqrt[3]{t} - \sqrt[3]{-t} \quad -1 \leq t \leq 3$$

$$\Rightarrow R f(t) = [f(-1), f(3)] \rightarrow \left[-\frac{1}{\sqrt[3]{2}}, \frac{1}{\sqrt[3]{2}} \right] \Rightarrow b - a = \frac{2}{\sqrt[3]{2}}$$

$$f(x) = x - [x + 2] \quad g(x) = \frac{x^2 - 2^{-x}}{x} \quad R g \circ f = ?$$

سؤال ١٠

$$f(x) = x - [x + 2] = x - [x] - 2 \quad \begin{cases} -2 & x \in \mathbb{Z} \\ -1 & x \notin \mathbb{Z} \end{cases}$$

$$\left(\frac{x^2 - 2^{-x}}{x} = -\frac{1}{x} \quad , \quad \frac{x^2 - 2^{-x}}{x} = -\frac{2}{x} \Rightarrow \right.$$

$$R g \circ f = \left[-\frac{1}{x}, -\frac{2}{x} \right)$$