

الف) $f(x) = \sqrt[3]{x^3 + x - 1}$ و تابع معکوس $f^{-1}(x)$ را بیابید. $f(x_1) = y_1$ و $f(x_2) = y_2$ $\Rightarrow$ $x_1^3 + x_1 - 1 = y_1^3$ و $x_2^3 + x_2 - 1 = y_2^3$ $\Rightarrow$ $x_1^3 - x_2^3 + x_1 - x_2 = y_1^3 - y_2^3$ $\Rightarrow$ $(x_1 - x_2)(x_1^2 + x_1x_2 + x_2^2 + 1) = (y_1 - y_2)(y_1^2 + y_1y_2 + y_2^2 + 1)$ $\Rightarrow$ $x_1 - x_2 = y_1 - y_2$ $\Rightarrow$ $x = y$ $\Rightarrow$ $f^{-1}(x) = x$ $\Rightarrow$ تابع همتی است.

ب) $x = \sqrt{y}$ $\Rightarrow$ $y = x^2$ $\Rightarrow$ $f(x) = x^2$ $\Rightarrow$ $f^{-1}(x) = \sqrt{x}$ $\Rightarrow$ $f(f^{-1}(x)) = x$ $\Rightarrow$ تابع همتی است.

ج) $\frac{x+y}{2} = \sqrt{xy}$ $\Rightarrow$ $(\sqrt{x})^2 + (\sqrt{y})^2 = 2(\sqrt{x}\sqrt{y})$ $\Rightarrow$ $(\sqrt{x} - \sqrt{y})^2 = 0$ $\Rightarrow$ $\sqrt{x} = \sqrt{y}$ $\Rightarrow$ $x = y$ $\Rightarrow$ تابع همتی است.

د) $xy^2 + x^2 - y^2 = 0$ $\Rightarrow$ $x(y^2 + x) - y^2 = 0$ $\Rightarrow$ $(y^2 + x)(x - y) = 0$ $\Rightarrow$ $x = y$ $\Rightarrow$ تابع همتی است.

الف) $x^2 = 2y - |y|$ $\Rightarrow$ $y \geq 0 \Rightarrow x^2 = y$ $\Rightarrow$ $y < 0 \Rightarrow x^2 = 2y - (-y) = 3y$ $\Rightarrow$ $3y > 0 \Rightarrow y > 0$ $\Rightarrow$ تناقض $\Rightarrow$ تابع همتی است.

ب) $[y] + [x] = 1$ $\xrightarrow{x=0.5}$ $[y] + [0.5] = 1$ $\Rightarrow$ $[y] = 1 - 0 = 1$ $\Rightarrow$ $1 \leq y < 2$ $\Rightarrow$ بازه ای که در آن y داده شده تابع همتی است.

$f(x) = \begin{cases} x^2 - b & x-1 \geq 2 \Rightarrow x \geq 3 \\ a + bx & x-1 < 2 \Rightarrow x < 3 \end{cases}$ $\xrightarrow{x=3}$ $f(3) = 3^2 - b = 3 + a$ $\Rightarrow$ $a + b = 18 - 6 = 12$ $\Rightarrow$ $a + b = 12$ $\Rightarrow$ تابع همتی است.

$f(x) = \sqrt{a-bx-x^2}$ $Op: [b, a]$ $f(a) = \sqrt{a-ab-a^2} = 0$ $f(b) = \sqrt{a-b^2-b^2} = 0$ $\Rightarrow$ $a(1-b-a) = 0$ $\Rightarrow$ $a = 0$ یا $a = 1-b-a$ $\Rightarrow$ $a = 1-b-a$ $\Rightarrow$ $2a = 1-b$ $\Rightarrow$ $a = \frac{1-b}{2}$ $\Rightarrow$ $a + b = \frac{1-b}{2} + b = \frac{1+b}{2}$ $\Rightarrow$ $a + b = 1$ $\Rightarrow$ تابع همتی است.

$f(x) = \sqrt{bx+c}$ $\Rightarrow$ $f(1) = \sqrt{b+c} = e$ $\Rightarrow$ $b+c = e^2$ $\Rightarrow$ $c = e^2 - b$ $\Rightarrow$ $f(x) = \sqrt{bx + e^2 - b}$ $\Rightarrow$ $f(1) = e$ $\Rightarrow$ $b + e^2 - b = e^2$ $\Rightarrow$ $e^2 = e^2$ $\Rightarrow$ تابع همتی است.

$$y = \sqrt{\frac{px+r}{x+b}} + a = \sqrt{\frac{(p+a)x+r+ab}{x+b}} \rightarrow \text{درجه دوم در صورت و مخرج همبستگی} \quad \begin{matrix} a = -p \\ b = -r \end{matrix}$$

$$\begin{array}{c|c} x & r \\ \hline p & + \\ f_{\text{lim}} & - \end{array} \rightarrow \underline{r+ab < 0}$$

(2)

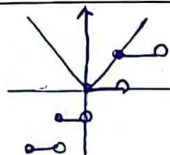
الف) $y = \sqrt{x^2 - x} \rightarrow x^2 - x \geq 0 \quad x(x-1) \geq 0 \quad x(x-1)(x+1) \geq 0$

$$\begin{array}{c|c} x & -1 \quad 0 \quad 1 \\ \hline p_{\text{lim}} & - \quad + \quad - \\ f_{\text{lim}} & + \quad - \quad + \end{array} \quad D_{f_{\text{lim}}} = [-1, 0] \cup [1, +\infty) \checkmark$$

(1)

ب) $y = \frac{1}{\sqrt{|x|-1}}$ $|x|-1 \neq 0 \quad |x| \neq 1 \quad x \neq \pm 1 \quad D_{f_{\text{lim}}} = \mathbb{R} - \{\pm 1\}$
 $|x|-1 > 0 \rightarrow |x| > 1 \rightarrow D = (-\infty, -1) \cup (1, +\infty)$

الف) $y = \sqrt{\frac{1}{|x| - [x]}}$ $\rightarrow |x| - [x] \neq 0 \quad |x| \neq [x] \quad x \neq 0, 1$



(1.5)

$D_{f_{\text{lim}}} = \mathbb{R} - \{0, 1\}$ همه اعداد صحیح مثبت و منفی حذف شوند
 ب) $\sqrt{x} + \sqrt{y} = 9 \rightarrow x \geq 0 \rightarrow \sqrt{y} = 9 - \sqrt{x} \quad 9 - \sqrt{x} \geq 0 \quad \sqrt{x} \leq 9 \quad x \leq 81$
 $\frac{II \cup I}{\cap} \rightarrow D_{f_{\text{lim}}} = [0, 81] \checkmark$

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$$y = \frac{1}{\sqrt{[x - \frac{1}{r}] + [x + \frac{p}{r}]}} \Rightarrow [x - \frac{1}{r}] + [x + \frac{p}{r}] \geq 0$$

$$[x - \frac{1}{r}] + [x - \frac{1}{r} + \frac{p}{r}] \geq 0 \quad r[x - \frac{1}{r}] + r \geq 0$$

$$[x - \frac{1}{r}] + 1 \geq 0 \quad [x - \frac{1}{r}] \geq -1 \quad x - \frac{1}{r} \geq 0 \quad x \geq \frac{1}{r} \quad D_{f_{\text{lim}}} = [\frac{1}{r}, +\infty) \checkmark$$

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$$y = \sqrt{1 - \log(x-a)} \quad x > \frac{a}{r} \quad 1 - \log(x-a) \geq 0 \quad \log(x-a) \leq 1$$

$$x \leq 1+a \quad x \leq \frac{a+1}{r} \quad \frac{a}{r} < x \leq \frac{a+1}{r} \rightarrow x \in (b, c]$$

$$c - b = \frac{(a+1)}{r} - \frac{a}{r} = \frac{1}{r} \checkmark$$

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(الف) $|n| - [n] \neq 0 \rightarrow |n| \neq [n] \rightarrow D = \mathbb{R} - \mathbb{N}$
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