

$$f(n) - n \geq -1 \Rightarrow f(n) \geq n-1$$

$$f(n) = \begin{cases} \sqrt{n-1} & n \geq 2 \\ \sqrt{n+2} & n < 2 \end{cases}$$

$$\left. \begin{aligned} \sqrt{n-1} \geq n-1 &\Rightarrow n \geq 0 \quad \checkmark \\ \sqrt{n+2} \geq n-1 &\Rightarrow \sqrt{n} \geq -\frac{3}{2} \Rightarrow n \geq -\frac{9}{4} \end{aligned} \right\}$$

$$\Rightarrow D_f = \left[-\frac{9}{4}, +\infty\right)$$

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$$f' = \frac{\sqrt{[12n^2 + 12n + 1]}}{(\sqrt{n+1})^4} = \frac{\sqrt{[12n^2 + 1 + 9n + 12n^2]}}{(\sqrt{n+1})^4} = \frac{\sqrt{(\sqrt{n+1})(\sqrt{n+2} + 1) + 9n(\sqrt{n+1})}}{(\sqrt{n+1})^4}$$

$$= \frac{\sqrt{(\sqrt{n+1})(\sqrt{n+2} + 1)}}{(\sqrt{n+1})^4} = \frac{\sqrt{(\sqrt{n+2} + 1)}}{(\sqrt{n+1})^3} \Rightarrow \sqrt{n+1} \neq 0 \Rightarrow n \neq -\frac{1}{4}$$

$$\Rightarrow D_f = \mathbb{R} - \left\{-\frac{1}{4}\right\}$$

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$$f(n) = \begin{cases} \frac{(n-1)(n^2 + \sqrt{n+2})}{(n-1)(n+1)} = \frac{(n-1)(n+1)(n+2)}{(n-1)(n+1)} = n+2 & ; n \neq \pm 1 \\ \frac{a+n^2}{n+b} & ; n = \pm 1 \end{cases}$$

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$$\Rightarrow c = \sqrt{2}$$

$$\sim \rightarrow n=1 \Rightarrow \frac{a+\sqrt{2}}{b+1} = \sqrt{2} \Rightarrow a+\sqrt{2} = \sqrt{2}b + \sqrt{2} \Rightarrow a - \sqrt{2}b = 0$$

$$n=-1 \Rightarrow \frac{\sqrt{2}-a}{b-1} = 1 \Rightarrow \sqrt{2}-a = b-1 \Rightarrow a+b = \sqrt{2}$$

$$\Rightarrow \begin{cases} a - \sqrt{2}b = 0 \\ a + b = \sqrt{2} \end{cases} \Rightarrow \begin{cases} b = 1/\sqrt{2} \\ a = \sqrt{2}/2 \end{cases}$$

$$c=1$$

$$n^2 + an + b = (n-1)^2 = n^2 - 2n + 1 \Rightarrow \begin{cases} a = -2 \\ b = 1 \end{cases} \Rightarrow a+b+c = -2+1+1 = 0$$

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$$\left. \begin{aligned} f(n+1) &= (n-2)^2(n) \\ f(1-a) &= (n+2)^2(-n) \end{aligned} \right\} \Rightarrow g(n) = (n-2)^2(n) - (n+2)^2(n) = n((n-2)^2 - (n+2)^2)$$

$$= -12n^2$$

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$$1-2n = -1 \Rightarrow n=1 \Rightarrow f(-1) = \sqrt{2}$$

$$1-2n = 3 \Rightarrow n=-1 \Rightarrow f(3) = 1 \Rightarrow f(-1) + f(3) = \sqrt{2} + 1 = \omega$$

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$$n + \sqrt{n} = 5 \Rightarrow n = 4 \Rightarrow f(4) = (\sqrt{4})^2 + \sqrt{4} = 4 + 2 = 6$$

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$$f(n-1) = (n-1)^2 + n-1 = n^2 - 2n^2 + 3n-1 + n-1 \Rightarrow f(n-1) + f(1-n) = 0$$

$$f(1-n) = (1-n)^2 + 1-n = -n^2 + 2n^2 - 3n+1 + 1-n$$

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$$\begin{cases} \text{هر دو نقطه} \Rightarrow b - \sqrt{2} = 0 \Rightarrow b = \sqrt{2} \\ n=1 \Rightarrow \sqrt{2} - \sqrt{2}a = 0 \Rightarrow \sqrt{2}a = \sqrt{2} \\ \text{تعریفی نشود} \Rightarrow \sqrt{2}f - g = \sqrt{2}k - 2 - \sqrt{2}k = k \Rightarrow k = -1 \end{cases} \Rightarrow \sqrt{2}a - \frac{b}{\sqrt{2}} - k = \sqrt{2} - 2 - (-1) = \sqrt{2}$$

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$$① (r_{n+r})(m-n) \geq 0 \Rightarrow \frac{-1}{-} + \frac{m}{-} \quad \text{D.f.} = [-1, r] \Rightarrow \underline{m=r}$$

$$② \xrightarrow{n=r} r+n-r\sqrt{r}=4\sqrt{r} \Rightarrow n=4\sqrt{r}-r \Rightarrow \underline{n+m=4\sqrt{r}+2}$$