

$$\begin{aligned} [a] > 2 &\leadsto a > \omega \quad \rightarrow \int \sqrt{r_{a-1}-a+1} = \sqrt{a} \leadsto a > 0 \quad \text{استند} \\ [a] < 2 &\leadsto a < \omega \quad \rightarrow \int \sqrt{r_{a+2}-a+1} : \sqrt{r_{a+2}} \rightarrow r_{a+2} > 0 \\ &\quad r_a > -2 \Rightarrow a > -\frac{4}{r} \\ &\quad \text{استند} \quad -\frac{4}{r} < a < \omega \\ \text{I} \cup \text{II} &\Rightarrow D_f : \left[-\frac{4}{r}, +\infty\right) \end{aligned}$$

$$\begin{aligned} f : \frac{r(1a^3 + 12a^2 + 6a + 1)}{(2a^2 + 6a + 1)^2} &= \frac{r(r_{a+1})^3}{(r_{a+1})^4} = \frac{r}{r_{a+1}} \Rightarrow \begin{cases} r_{a+1} \neq 0 \\ r_a \neq -1 \\ a \neq -\frac{1}{r} \end{cases} \\ &\Rightarrow D_f : \mathbb{R} - \left\{-\frac{1}{r}\right\} \end{aligned}$$

$$\begin{aligned} a=0 &\leadsto f(0) = \frac{-r}{-1} = r, \quad g(0) = 0 + C = C \Rightarrow \boxed{r=C} \\ a=1 &\leadsto f(1) = \frac{a+r}{1+b}, \quad g(1) = 1+r, \quad \Rightarrow \frac{a+r}{1+b} = r \leadsto a+r = r+rb \\ a=-1 &\leadsto f(-1) = \frac{r-a}{b-1}, \quad g(-1) = 1 \Rightarrow \frac{r-a}{b-1} = 1 \leadsto r-a = b-1 \\ &\quad a+r, r-a = r+rb+b-1 \\ &\quad r : rb+r \rightarrow b = \frac{1}{r} \\ \Rightarrow b+C &= \frac{1}{r} + r = r, \omega \end{aligned}$$

$$\begin{aligned} g(n) &= \frac{(n-1)(n+1)}{a^2 + an + b} \rightarrow \begin{aligned} &\text{اگر } a=1 \text{ باشد، راجع به } n \text{ درجه دوم باشد. پس باید درجه دوم باشد.} \\ &\text{و } [C] \text{ به } f \text{ تعلق نداشته باشد، پس درجه دومی گزینیم.} \end{aligned} \\ \Rightarrow g(n) &= \frac{(n-1)(n+1)}{(n-C)(n-1)} \leadsto D_f \neq \mathbb{R} - \{C, 1\} \quad D_f : \mathbb{R} - \{C\} \\ &\quad C=1 \Leftrightarrow \begin{aligned} &a = -\frac{r}{2} \\ &b = \frac{1}{2} \\ &c = 1 \end{aligned} \Rightarrow a+b+C = -\frac{r}{2} + 1 + 1 = 0 \end{aligned}$$

$$\begin{aligned} f(n+1) &= (n-r)^r(n), \quad f(-n+1) = (-n-r)^r(-n) \\ \rightarrow f(n+1) + f(-n+1) &= (n-r)^r(n) + (-n-r)^r(-n) = (n) \left\{ (n-r)^r + (-1)(-n-r)^r \right\} \\ &= (n) \left\{ n^r - 2nr + 2 - n^r - 2nr - 2 \right\} = (n)(-4n) = -4n^2 \end{aligned}$$

$$f(1) \leadsto 1 - \sqrt{m} = -1 \rightarrow m = 1 \quad \leadsto f(-1) = \sqrt{1} = 1$$

$$f(r) \leadsto 1 - \sqrt{m} = r \rightarrow m = -1 \quad \leadsto f(r) = \sqrt{-1} + r = 1$$

$$\Rightarrow f(-1) + f(r) = r + 1 = \omega$$

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$$f(4) \leadsto m + \sqrt{m} = 4 \rightarrow m + \sqrt{m} - 4 = 0 \Rightarrow t^2 + t - 4 = 0$$

$$(t+r)(t-r) = 0 \quad \begin{cases} t=r \Rightarrow \sqrt{m}=r \\ t=-r \Rightarrow \sqrt{m}=0 \end{cases}$$

$$\Rightarrow m = 2$$

$$\leadsto f(4) = \sqrt{2} + \sqrt{2} = 2\sqrt{2}$$

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$$f(n) = n^r + n = n(n^{r-1} + 1)$$

$$\rightarrow f(n-1) = (n-1)(n^{r-1} - r + 1 + 1)$$

$$\rightarrow f(1-n) = (1-n)(n^{r-1} - r + 1 + 1)$$

$$\rightarrow f(n-1) + f(1-n) = (n^{r-1} - r + 2)(n-1+1-n)$$

$$\Rightarrow f(n-1) + f(1-n) = 0$$

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$$D_f: m \geq \frac{r}{2}a, \quad D_g: m \leq \frac{b}{2} \rightarrow D_f \cap D_g: \{1\}$$

$$\rightarrow \frac{r}{2}a = \frac{b}{2} = 1 \Rightarrow b = r, \quad a = \frac{2}{r}$$

$$r f(1) - g(1) = r(1-r) - rk = -k-r \rightarrow -k-r = k \Rightarrow -r = 2k \Rightarrow k = -1$$

$$ra - \frac{b}{r} - k = r\left(\frac{2}{r}\right) - \frac{r}{r} - (-1) = r - r + 1 = 1$$

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$$D_f: m \leq m, \quad D_g: m \geq -1 \rightarrow D_{f \cap g}: [-1, m] \Rightarrow m = v$$

$$f(r) = \sqrt{v-r} + n = r + n \quad g(r) = \sqrt{v} = r + \sqrt{r}$$

$$\rightarrow f(r) - g(r) = r + n - r - \sqrt{r} = 0 \Rightarrow n = \sqrt{r} - r$$

$$\Rightarrow m + n = v + \sqrt{r} - r = \omega + \sqrt{r}$$

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