

$$f(n) - n + 1 \geq 0 \rightarrow f(n) + 1 \geq n$$

$$f(n) \rightarrow 2n-1; n \geq 0 \rightarrow f(n)+1 \geq n \xrightarrow{n \geq 0} 2n-1+1-n \geq 0 \rightarrow n \geq 0 \xrightarrow{(1)} n \geq 0$$

$$\rightarrow f(n) \rightarrow 2n+3; n < 0$$

$$\xrightarrow{(1)} -\frac{3}{2} \leq n < 0 \quad Dg = I \cup II = -\frac{3}{2} \leq n \left[-\frac{3}{2}, +\infty \right)$$

$$y = \frac{2f(n)^2 + 3gn^2 + 1(n+1)^2}{(f(n)^2 + 2n+1)^2} = \frac{2(4n^2 + 12n^2 + 4n+1)}{(2n+1)^4} = \frac{2(2n+1)^2}{(2n+1)^4} = \frac{2}{2n+1}$$

$$Dg = \mathbb{R} - \left\{ -\frac{1}{2} \right\}$$

$$f(n) = \begin{cases} \frac{n^2 + 2n^2 - n - 2}{(n-1)(n+1)} = \frac{(n-1)(n^2 + 2n+2)}{(n-1)(n+1)} = \frac{(n-1)(n+2)(n+1)}{(n-1)(n+1)} = n+2 \end{cases}$$

$$g(n) = n + c \xrightarrow{\frac{a+n}{x+b}} \begin{cases} n=1 \rightarrow \frac{a+1}{1+b} = 2 \rightarrow a+1 = 2(1+b) \rightarrow a = 2b+1 \\ n=-1 \rightarrow \frac{-a+1}{b-1} = 1 \rightarrow -a+1 = b-1 \rightarrow a+b = 2 \end{cases}$$

$$\rightarrow n+c = n+2 \Rightarrow c=2 \Rightarrow \begin{cases} n=1 \rightarrow y=2 \\ n=-1 \rightarrow y=1 \end{cases} \rightarrow \begin{cases} a=2b+1 \\ a+b=2 \end{cases} \rightarrow \begin{cases} b=\frac{1}{3} \\ a=\frac{5}{3} \end{cases}$$

$$\left. \begin{aligned} f(n) &= \frac{n+1}{n-c} \\ g(n) &= \frac{(n-1)(n+1)}{n^2 + an + b} \end{aligned} \right\} \rightarrow n^2 + an + b = (n-c)(n-1) \xrightarrow{Dg=Df} (n-1) = (n-c)$$

$$\Rightarrow n^2 + an + b = (n-1)^2 = n^2 - 2n + 1$$

$$\rightarrow \begin{cases} a = -2 \\ b = 1 \\ c = 1 \end{cases} \rightarrow a+b+c = (-2)+1+1 = 0$$

$$f(n) = (n-2)^2(n-1)$$

$$f(n+1) + f(1-n) = (n-2)^2 n + (-n-2)^2(-n) = (n-2)^2 n - n(n+2)^2$$

$$n^3 - 4n^2 + 8n - n^3 - 4n^2 - 8n = -8n^2$$

$$1-2n=3 \rightarrow n=-1 \rightarrow \sqrt[n]{n}+2 = \textcircled{1}$$

$$1-2n=-1 \rightarrow n=1 \rightarrow \sqrt[n]{n} = \textcircled{4}$$

$$\left. \begin{array}{l} \textcircled{1} \\ \textcircled{4} \end{array} \right\} \rightarrow f(-1)+f(3)=1+4=\textcircled{5}$$

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$$n+\sqrt{n}=4 \rightarrow n-4=\sqrt{n} \xrightarrow{\text{توان}} n^2-12n+16=n \rightarrow n^2-13n+16=0$$

$$(n-2)(n-8)=0 \rightarrow n \in \{2, 8\}$$

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$$f(4+\sqrt{4})=f(4)=(4)^4+3(4)=\textcircled{244}$$

$$f(n-1)+f(1-n)=(n-1)^3+(n-1)+(1-n)^3+(1-n)=\textcircled{\text{صفر}}$$

قرینه هم بنابرین حذف می شوند

قرینه اند بنابرین جمع آنها صفر می شود.

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$$f(x)=\sqrt{x-3a}+k-1 \rightarrow D_f=\left[\frac{3a}{2}, +\infty\right)$$

$$g(x)=\sqrt{b-3x}+3k \rightarrow D_g=(-\infty, +\frac{b}{3}]$$

$$D_f \cap D_g = \{ \} \rightarrow \frac{3a}{2} = +\frac{b}{3}$$

$$3a = +b$$

$$f-g=(1, k) \rightarrow 2(\sqrt{x-b}+k-1) - (\sqrt{b-x}+3k) = 2\sqrt{x-b}+2k-2-\sqrt{b-x}-3k=k$$

$$\rightarrow 2k=-2 \rightarrow k=-1$$

$$3a=b \rightarrow a=\frac{5}{3}$$

$$3a - \frac{b}{3} - k = 3\left(\frac{5}{3}\right) - \frac{5}{3} - (-1) = \textcircled{3}$$

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$$f(x)=\sqrt{m-x}+n \rightarrow D_f=(-\infty, m]$$

$$g(x)=\sqrt{2x+1} \rightarrow D_g=[-1, +\infty)$$

$$D_f \cap D_g = [-1, m] = [-1, 7] \rightarrow m=7$$

$$\rightarrow f(x)=\sqrt{7-x}+n$$

$$f(3)=\sqrt{7-(3)}+n$$

$$g(3)=\sqrt{2(3)+1}=\sqrt{7}$$

$$\rightarrow f-g=2+n-\frac{2\sqrt{7}}{\sqrt{7}}=4\sqrt{7} \rightarrow n=8\sqrt{7}-2$$

$$m+n=7+8\sqrt{7}-2=\textcircled{5+8\sqrt{7}}$$

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