

۱۹, ۵

$$f(x) = \begin{cases} 2x-1 & x \geq 5 \\ 5x+3 & x < 5 \end{cases}$$

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$$g = \begin{cases} \sqrt{x} & x \geq 5 \rightarrow x \geq 0 \xrightarrow{\cap} x \geq 5 \quad A \\ \sqrt{3x+4} & x < 5 \rightarrow x \geq -\frac{4}{3} \xrightarrow{\cap} -\frac{4}{3} \leq x < 5 \quad B \end{cases}$$

$$A \cap B \rightarrow D_f = \left[-\frac{4}{3}, +\infty \right)$$

$$\frac{25x^3 + 24x^2 + 11x + 3}{(5x^2 + 4x + 1)^2} = \frac{4x+3}{5x^2 + 4x + 1} = \frac{3(2x+1)}{(2x+1)^2} - 2 = \frac{3}{2x+1}$$

ساده شده عبارت هم باید در نوشتن

$$\rightarrow 5x^2 + 4x + 1 \neq 0 \Rightarrow x \neq -\frac{1}{5} \Rightarrow D_f = \mathbb{R} - \left\{ -\frac{1}{5} \right\}$$

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$$\frac{x^3 + 2x^2 - x - 2}{x^2 - 1} = \frac{(x-1)(x^2 + 3x + 2)}{(x-1)(x+1)}$$

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$$= \frac{x^2 + 3x + 2}{x+1} = \frac{(x+1)(x+2)}{x+1} = x+2 \Rightarrow C=2$$

$$\begin{cases} 1+2=3 \Rightarrow \frac{a+2}{b+1} = 3 \Rightarrow a-3b=1 \\ -1+2=1 \Rightarrow \frac{-a+2}{b-1} = 1 \Rightarrow a+b=3 \end{cases} \Rightarrow \begin{cases} a=2, b=1 \end{cases}$$

$$\Rightarrow a+b+C = \boxed{5}$$

$$D_f = D_g \Rightarrow x^2 + ax + b = (x-c)^2$$

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$$= x^2 - 2cx + c^2 \Rightarrow a = -2c, b = c^2$$

$$\rightarrow \frac{x^2-1}{(x-1)^2} = \frac{(x-1)(x+1)}{(x-1)^2} \Rightarrow x-c = x-1 \Rightarrow c=1$$

$$\Rightarrow \begin{cases} c=1 \\ b=1 \\ a=-2 \end{cases}$$

$$a+b+C = \boxed{0}$$

$$f(x+1) = (x-2)^2(x)$$

$$f(1-x) = (-2-x)(-x)$$

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$$\begin{aligned} 1 \Rightarrow f(x+1) + f(1-x) &= (x-2)^2(x) + (x+2)^2(-x) \\ &= x((x-2)^2 - (x+2)^2) = x(-4x) = -4x^2 \\ \Rightarrow g(x) &= -4x^2 \end{aligned}$$

$$1 - 2x = -1 \Rightarrow x = 1 \Rightarrow f(-1) = 5 \checkmark$$

$$1 - 2x = 3 \Rightarrow x = -1 \Rightarrow f(3) = 1 \checkmark$$

$$\Rightarrow f(-1) + f(3) = 5 + 1 = 6 \checkmark$$

$$y = x + \sqrt{x} \Rightarrow x = 4 \Rightarrow f(4) = 2 \Delta y + 1 = 9 \checkmark$$

$$x - 1 = -(1 - x) \Rightarrow f(x - 1) = -f(1 - x)$$

$$\Rightarrow f(x - 1) + f(1 - x) = 0 \checkmark$$

$$D_{f-g} = \{1\}$$

$$\Rightarrow \begin{cases} x - a \geq 0 \Rightarrow x \geq \frac{a}{f} \rightarrow \frac{a}{f} = 1 \Rightarrow a = f \\ b - x \geq 0 \Rightarrow x \leq \frac{b}{f} \rightarrow \frac{b}{f} = 1 \Rightarrow b = f \end{cases}$$

$$\Rightarrow \begin{cases} f(x) = k - 1 \\ g(x) = k \end{cases} x = 1$$

$$\Rightarrow 2f - g = 2k - 1 - k = k \Rightarrow k = -1$$

$$\Rightarrow 2a - \frac{b}{f} - k = 5 - 1 - (-1) = 5 \checkmark$$

$$f(x) = \sqrt{m-x} + n$$

$$g(x) = \sqrt{x} = \sqrt{x}$$

$$\Rightarrow f - g(x) = \sqrt{m-x} + n - \sqrt{x} = 4\sqrt{x}$$

$$\Rightarrow \sqrt{m-x} + n = 4\sqrt{x}$$

$$D_{f \times g} = [-1, 5] \Rightarrow \begin{cases} m - x \geq 0 \Rightarrow x \leq m \Rightarrow m = 5 \\ 2x + 1 \geq 0 \Rightarrow x \geq -1 \end{cases}$$

$$f - g(x) = 2 + n - 2\sqrt{x} = 4\sqrt{x}$$

$$\Rightarrow 2 + n = 4\sqrt{x} \Rightarrow n = 4\sqrt{x} - 2$$

$$\Rightarrow m + n = 5 + 4\sqrt{x} - 2 = 3 + 4\sqrt{x} \checkmark$$