

آفرین!

$$y = \sqrt{f(x) - x + 1}$$

$$f(x) = \begin{cases} x^2 - 1 : [x] > 4 \Rightarrow x \geq 5 \rightarrow y = \sqrt{x^2 - 1 - x + 1} = \sqrt{x} \rightarrow x \geq 0 \\ x^2 + 3 : [x] \leq 4 \Rightarrow x < 5 \rightarrow y = \sqrt{x^2 + 3 - x + 1} = \sqrt{x^2 + 4} \rightarrow x^2 + 4 \geq 0 \\ [-\frac{4}{x}] = -2 \leq 4 \Rightarrow D_f = [-\frac{4}{x} \cup +\infty) \end{cases}$$

$$y = \frac{x^2 x^2 + 4x^2 + 1x + 3}{(x^2 + x + 1)^2} = \frac{3(1x^2 + 1x^2 + 4x + 1)}{(x^2 + x + 1)^2} = \frac{3(2x + 1)}{(x^2 + x + 1)^2} = \frac{3}{x^2 + x + 1}$$

$$\rightarrow D_f = \mathbb{R} - \{-\frac{1}{2}\}$$

$$f(x) = \frac{x^2 + 2x^2 - x^2}{x^2 - 1} : x \neq \pm 1$$

$$f(x) = \begin{cases} \frac{x^2 + 2x^2 - x^2}{x^2 - 1} : x \neq \pm 1 \\ \frac{a x^2}{x + b} : x = \pm 1 \end{cases}$$

$$f(0) = g(0) = 2 = 0 + c \Rightarrow c = 2$$

$$x = 1 \Rightarrow \frac{a + 2}{1 + b} = 1 + c \Rightarrow 1 + b + c + b c = a + 2$$

$$x = -1 \Rightarrow \frac{1 - a}{b - 1} = c - 1 \Rightarrow 1 - b - c + b c = -a + 2$$

$$g(x) = x + c$$

$$b + c = \frac{1}{2} + 2 = \frac{5}{2}$$

$$b \times c = 1 \Rightarrow b = \frac{1}{c}$$

$$\frac{1}{c} + c = \frac{5}{2} \Rightarrow \frac{1 + c^2}{c} = \frac{5}{2} \Rightarrow 2 + 2c^2 = 5c \Rightarrow 2c^2 - 5c + 2 = 0 \Rightarrow c = 2 \text{ or } c = \frac{1}{2}$$

$$f(x) = \frac{x+1}{x-c}$$

$$f(x) = g(x) \Rightarrow D_{f(x)} = D_{g(x)} \quad b=c, a=-c-1$$

$$c=1$$

$$g(x) = \frac{x^2 - 1}{x^2 + a x + b} \Rightarrow \frac{(x+1)(x-1)}{(x-1)(x-c)} = \frac{x+1}{x^2 - (a+1)x + c}$$

$$c=1, b=1, a=-2 \Rightarrow D_g \neq 1, c \text{ و } D_f \neq c \Rightarrow c=1$$

$$a + b + c = 1 + 1 - 2 = 0$$

$$f(x) = (x-3)^2(x-1)$$

$$x+1 < x+1 \quad 1-x < 1-x-1$$

$$g(x) = f(x+1) + f(1-x) = (x-2)^2(x) + (2-x)^2(-x)$$

$$(x^2 - 4x + 4)(x) - (x^2 + 4x + 4)(-x)$$

$$= x(x^2 - 4x + 4 - x^2 - 4x - 4) \Rightarrow g(x) = -8x$$

$f(1-\sqrt{2}) = \begin{cases} \sqrt{2}+1 & x \leq 0 \\ \sqrt{2} & x > 0 \end{cases}$ $f(-1) + f(\sqrt{2}) =$ $\Downarrow \quad \Downarrow$ $\sqrt{2} + \sqrt{-1} + 1 = \boxed{2} \checkmark$	2
$f(x+\sqrt{x}) = x^2 + \sqrt{x}$ $f(4) = ? = (\sqrt{4})^2 + \sqrt{4} = 2 + 2 = \boxed{4} \checkmark$ $x + \sqrt{x} = 4$ $x + \sqrt{x} - 4 = 0 \xrightarrow{\sqrt{x}=t} t^2 + t - 4 = 0$ $(t+5)(t-4) \Rightarrow t = \begin{cases} \sqrt{x} = 4 \checkmark \Rightarrow x = 16 \\ \sqrt{x} = -5 \end{cases}$	2
$f(x) = x^2 + x$ $f(x-1) + f(1-x) = (x-1)^2 + (x-1) + -(x-1)^2 + (1-x)$ $= \boxed{0} \checkmark$	1
$f(x) = \sqrt{x^2 - 4x} + k - 1$ $g(x) = \sqrt{b - 4x} + k$ $D_f = \{x^2 - 4x \geq 0, x \geq \frac{4}{4}\}$ $D_g = \{b - 4x \geq 0, x \leq \frac{b}{4}\}$ $D_{f \cap g} = \left\{ \frac{4}{4} \leq x \leq \frac{b}{4} \right\} \Rightarrow \frac{4}{4} = \frac{b}{4} \Rightarrow 4 = b$ $f - g(1) = \sqrt{1^2 - 4 \cdot 1} + k - 1 - (\sqrt{b - 4 \cdot 1} + k) = k - 1 - \sqrt{b - 4} - k = -1 - \sqrt{b - 4}$ $-1 - \sqrt{b - 4} = -1 \Rightarrow \sqrt{b - 4} = 0 \Rightarrow b - 4 = 0 \Rightarrow b = 4$	2
$f(x) = \sqrt{m-x} + n \quad x \leq m$ $g(x) = \sqrt{x+1} \rightarrow x \geq -1$ $D_{g \cap f} = [-1, m] = D_f \cap D_g \Rightarrow m = \sqrt{1} = 1$ $f - g(1) = \sqrt{m-1} + n - \sqrt{1+1} = \sqrt{m-1} + n - \sqrt{2} = 4\sqrt{2}$ $\sqrt{m-1} + n - \sqrt{2} = 4\sqrt{2} \Rightarrow \sqrt{m-1} + n = 5\sqrt{2}$ $m+n = 1 + 4\sqrt{2} \Rightarrow n = 4\sqrt{2} - 1$	2