

۱۹, ۵

A دوازدهم سپهر

کریستین شاپور

$$f(x) = \begin{cases} 2x-1 & x \geq \omega \rightarrow y = \sqrt{x} \rightarrow x \geq 0 \rightarrow D = [\omega + \infty) \text{ I - 1} \\ 2x+3 & x < \omega \rightarrow y = \sqrt{3x+6} \rightarrow x \geq -\frac{6}{3} \rightarrow D = [-\frac{6}{3}, \omega) \text{ II (2)} \end{cases}$$

$$D_f = I \cup II = [-\frac{6}{3}, +\infty) \checkmark$$

$$y = \frac{3(2x^3 + 2x^2 + 4x + 1)}{(5x^2 + 5x + 1)^2}$$

$$\rightarrow y = \frac{3(2x+1)(5x^2 - 2x + 1) + 4x(2x+1)}{(5x^2 + 5x + 1)^2}$$

$$\rightarrow y = \frac{3(2x+1)(5x^2 + 5x + 1)}{(5x^2 + 5x + 1)^2}$$

$$\rightarrow y = \frac{3(2x+1)}{(2x+1)^2} \rightarrow y = \frac{3}{2x+1} \checkmark$$

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$$(2x+1) \neq 0$$

$$\rightarrow 2x+1 \neq 0$$

$$\rightarrow x \neq -\frac{1}{2}$$

$$\rightarrow D_f = \mathbb{R} - \{-\frac{1}{2}\}$$

$$\frac{x^3 - x + 2x^2 - 2}{x^2 - 1}$$

$$\rightarrow \frac{(x)(x^2-1) + 2(x^2-1)}{x^2-1}$$

$$\rightarrow \frac{(x^2-1)(x+2)}{x^2-1} = x+2 \Rightarrow x+2 = x+2 \Rightarrow x=0 \checkmark$$

$$x+2 = \frac{ax+b}{x+b} \xrightarrow{x=1} 1+2 = \frac{a+b}{1+b} \Rightarrow 3 = \frac{a+b}{1+b}$$

$$x+2 = \frac{ax+b}{x+b} \xrightarrow{x=-1} -1+2 = \frac{-a+b}{-1+b} \Rightarrow 1 = \frac{-a+b}{-1+b} \Rightarrow b-a = 1 \Rightarrow b = a+1$$

$$\rightarrow b+a = 2 \Rightarrow 2+0 = 2 \Rightarrow \frac{2}{1} \checkmark$$

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۴- چون در دامنه فقط یکی عدد وجود ندارد پس $\frac{(x-1)(x+1)}{x^2+9x+6}$ را به صورت $\frac{x+1}{x-c}$ می‌نویسیم و باید ریشه‌های x^2+9x+6 را پیدا کنیم

$$\frac{(x-1)(x+1)}{x^2+9x+6} = \frac{x+1}{x-c}$$

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$$\Rightarrow (x-1)^2 = x^2+9x+6 \Rightarrow x^2-2x+1 = x^2+9x+6$$

$$\Rightarrow \begin{cases} a = -9 \\ b = 1 \end{cases}$$

$$\frac{(x-1)(x+1)}{(x-1)^2} = \frac{x+1}{x-c} \Rightarrow x-1 = x+1 \Rightarrow c = -1$$

$$a+b+c = -9+1+1 = -7$$

$$f(x+1) = (x+1-3)^2(x+1-1) = (x-2)^2(x)$$

$$f(1-x) = (1-x-3)^2(1-x-1) = (x+2)^2(-x)$$

$$\Rightarrow g(x) = (x-2)^2(x) - (x+2)^2(-x)$$

$$\rightarrow g(x) = x(x^2-4x+4 - x^2-4x-4)$$

$$\rightarrow g(x) = -8x^2$$

$$1-2x = -1 \Rightarrow x = 1 \rightarrow f(1) = 1 \cdot 1 = 1$$

$$x = 1-2x \Rightarrow x = -1 \rightarrow f(-1) = 1 \cdot 1 = 1$$

$$f(-1) + f(1) = 1 + 1 = 2$$

$$x + \sqrt{x} = 4 \rightarrow x + \sqrt{x} - 4 = 0 \rightarrow (\sqrt{x}+3)(\sqrt{x}-2) = 0 \Rightarrow \begin{cases} \sqrt{x} = -3 \rightarrow x = -9 \\ \sqrt{x} = 2 \rightarrow x = 4 \end{cases}$$

$$f(4) = (4)^2 + 3(4) = 16 + 12 = 28$$

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$$f(x-1) = (x-1)^3 + x - 1$$

$$f(1-x) = (1-x)^3 - x + 1 = -(x-1)^3 - x + 1$$

$$\Rightarrow f(x-1) + f(1-x) = (x-1)^3 + x - 1 - (x-1)^3 - x + 1 = 0$$

۹- چون f و g فقط دارای یک زوج مرتب (a, b) باشد نتیجه می گیریم که تنها اشتراک دامنه های f و g همان است در نتیجه

$$\left. \begin{aligned} 4x - 2a \geq 0 &\rightarrow x \geq \frac{2a}{4} \\ 6 - 4x \geq 0 &\rightarrow x \leq \frac{6}{4} \end{aligned} \right\} \Rightarrow \frac{2a}{4} = \frac{6}{4} = 1 \Rightarrow \boxed{a = 2} \quad \boxed{b = 3}$$

$$f \circ g = 2 \left(\sqrt{4 - 4(1)} + 3k \right) - \left(\sqrt{4 - \left(\frac{2}{4}\right)^3} + k - 1 \right) = k$$

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$$\Rightarrow 2k - k + 1 = k \Rightarrow \boxed{k = 1}$$

$$2a - \frac{b}{2} - k = 2\left(\frac{2}{2}\right) - \frac{3}{2} - \left(-\frac{1}{2}\right) = 1$$

$$2x + 2 > 0$$

$$\rightarrow x > -1$$

$$m - x \geq 0 \rightarrow x \leq m \quad \left. \begin{aligned} &\rightarrow x > -1 \\ &\rightarrow x \leq m \end{aligned} \right\} D_{f \circ g} = [-1, m]$$

$$m = 1$$

$$f \circ g(1) = 2\sqrt{2} \rightarrow \sqrt{4 - 3} + n - \sqrt{4 - \frac{1}{4}} = 2\sqrt{2}$$

$$\Rightarrow 1 + n - \sqrt{2} = 2\sqrt{2} \Rightarrow n = \sqrt{2} + 1$$

$$m + n = 1 + \sqrt{2} + 1 = 2 + \sqrt{2}$$

$$D_f = \left[\frac{2a}{4}, +\infty\right) \quad D_g = (-\infty, \frac{b}{4}] \quad \Delta \rightarrow \{1\}$$

۹- اشتراک دامنه f و g فقط $a = 2$ است پس:

$$\frac{2a}{4} = \frac{b}{4} = 1 \rightarrow a = 2 \quad b = 4$$

$$2f(1) - g(1) = k \rightarrow 2k - 2 - 3k = k \rightarrow k = -1$$

$$2a - \frac{b}{2} - k = 2 \cdot 2 - \frac{4}{2} - (-1) = 1$$