

$$f(x) = \begin{cases} 2x-1 & x \geq \omega \rightarrow y = \sqrt{x} \rightarrow x \geq 0 \rightarrow D = [\omega + \infty) \text{ I - I} \\ 2x+3 & x < \omega \rightarrow y = \sqrt{3x+4} \rightarrow x \geq -\frac{4}{3} \rightarrow D = [-\frac{4}{3}, \omega) \text{ II} \end{cases}$$

$$D_f = I \cup II = [-\frac{4}{3}, +\infty)$$

$$y = \frac{3(2x^3 + 2x^2 + 4x + 1)}{(5x^2 + 5x + 1)^2}$$

$$\rightarrow y = \frac{3(2x+1)(5x^2 - 2x + 1) + 4x(2x+1)}{(5x^2 + 5x + 1)^2}$$

$$\rightarrow y = \frac{3(2x+1)(5x^2 + 5x + 1)}{(5x^2 + 5x + 1)^2}$$

$$\rightarrow y = \frac{3(2x+1)}{(5x+1)^2} \rightarrow y = \frac{3}{2x+1}$$

$$(2x+1) \neq 0$$

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$$\rightarrow x \neq -\frac{1}{2}$$

$$\rightarrow D_f = \mathbb{R} - \{-\frac{1}{2}\}$$

$$\frac{x^3 - x + 2x^2 - 2}{x^2 - 1}$$

$$\rightarrow \frac{(x)(x^2-1) + 2(x^2-1)}{x^2-1}$$

$$\rightarrow \frac{(x^2-1)(x+2)}{x^2-1} = x+2 \Rightarrow x+2 = x+2 \Rightarrow x=0$$

$$x+2 = \frac{ax+b}{x+b} \xrightarrow{x=1} 3 = \frac{a+b}{1+b} \Rightarrow 3(1+b) = a+b \Rightarrow 3+3b = a+b \Rightarrow 3+2b = a$$

$$x+2 = \frac{ax+b}{x+b} \xrightarrow{x=-1} 1 = \frac{-a+b}{-1+b} \Rightarrow 1(-1+b) = -a+b \Rightarrow -1+b = -a+b \Rightarrow -1 = -a \Rightarrow a=1$$

$$\rightarrow b+c \rightarrow 2+0/2 = 2/2$$

۴- چون در دامنه فقط یکی عدد وجود ندارد پس $\frac{(x-1)(x+1)}{x^2+9x+6}$ را به صورت $\frac{a}{x-1} + \frac{b}{x+1} + \frac{c}{x-2}$ می‌نویسیم و باید ریشه‌های مخرج باشد در نتیجه

$$\frac{(x-1)(x+1)}{x^2+9x+6} = \frac{x+1}{x-2}$$

$$\Rightarrow (x-1)^2 = x^2+9x+6 \Rightarrow x^2-2x+1 = x^2+9x+6$$

$$\Rightarrow \begin{cases} a = -7 \\ b = 1 \end{cases}$$

$$\frac{(x-1)(x+1)}{(x-1)^2} = \frac{x+1}{x-1} \Rightarrow x-1 = x-1 \Rightarrow \boxed{c=1}$$

$$a+b+c = -7+1+1 = \textcircled{0}$$

$$f(x+1) = (x+1-3)^2 (x+1-1) = (x-2)^2 (x)$$

$$f(1-x) = (1-x-3)^2 (1-x-1) = (x+2)^2 (-x)$$

$$\Rightarrow g(x) = (x-2)^2 (x) - (x+2)^2 (-x)$$

$$\rightarrow g(x) = x(x^2-4x+4 - x^2-4x-4)$$

$$\rightarrow \boxed{g(x) = -8x^2}$$

$$1-2x = -1 \Rightarrow x = 1 \rightarrow f(1) = \sqrt{1} = 1$$

$$x = 1-2x \Rightarrow x = -1 \rightarrow f(-1) = \sqrt{-1+4} = 1$$

$$f(-1) + f(x) = 1 + 1 = \textcircled{2}$$

$$x + \sqrt{x} = 4 \rightarrow x + \sqrt{x} - 4 = 0 \rightarrow (\sqrt{x}+3)(\sqrt{x}-2) = 0 \begin{cases} \sqrt{x} = -3 \rightarrow x = -9 \\ \sqrt{x} = 2 \rightarrow x = 4 \end{cases}$$

$$f(4) = (4)^2 + 3(4) = 16 + 12 = \boxed{28}$$

$$f(x-1) = (x-1)^3 + x - 1$$

$$f(1-x) = (1-x)^3 - x + 1 = -(x-1)^3 - x + 1$$

$$\Rightarrow f(x-1) + f(1-x) = (x-1)^3 + x - 1 - (x-1)^3 - x + 1 = 0$$

۹- چون f و g فقط دارای یک زوج مرتب $(1, k)$ باشد نتیجه می گیریم که تنها اشتراک دامنه های f و g این است در نتیجه

$$\left. \begin{aligned} 4x - 2a > 0 &\rightarrow x > \frac{2a}{4} \\ 6 - 4x > 0 &\rightarrow x < \frac{6}{4} \end{aligned} \right\} \Rightarrow \frac{2a}{4} = \frac{6}{4} = 1 \Rightarrow \boxed{a = \frac{2}{1}} \quad \boxed{b = \frac{6}{1}}$$

$$f \circ g = 2 \left(\sqrt{4 - \frac{6}{4}} + 3k \right) - \left(\sqrt{4 - \left(\frac{2}{4} \right)^3} + k - 1 \right) = k$$

$$\Rightarrow 2k - k + 1 = k \Rightarrow \boxed{k = -\frac{1}{4}}$$

$$3a - \frac{6}{4} - k = 3 \left(\frac{2}{4} \right) - \frac{6}{4} - \left(-\frac{1}{4} \right) = \frac{9}{4}$$

۱۰- $D_{f \circ g}[-1, \sqrt{2}] = 1$ در واقع اشتراک بین دو تابع f و g را نشان می دهد

$$\left. \begin{aligned} 2x + 2 > 0 \\ \rightarrow x > -1 \\ m - x \geq 0 \rightarrow x \leq m \end{aligned} \right\} D_{f \circ g}[-1, \sqrt{2}] \quad m = \sqrt{2}$$

$$f \circ g(x) = 2\sqrt{2} \rightarrow \sqrt{2 - 3} + n - \sqrt{2 - 3} = 2\sqrt{2}$$

$$\Rightarrow 2 + n - 2\sqrt{2} = 2\sqrt{2} \Rightarrow n = 4\sqrt{2} - 2$$

$$m + n = \sqrt{2} + 4\sqrt{2} - 2 = \boxed{5\sqrt{2} - 2}$$