

دامنه و بازه تعریف تابع: $x \geq 0$

$$f(x) \geq x-1$$

$$f(x) = \begin{cases} 2x-1, & x \geq 0 \\ 5x+3, & x < 0 \end{cases}$$

$$2x-1 \geq x-1 \rightarrow x \geq 0 \xrightarrow{x \geq 0} \text{I: } \boxed{x \geq 0}$$

$$5x+3 \geq x-1 \rightarrow x \geq -\frac{4}{4} \xrightarrow{x < 0} \text{II: } \boxed{-\frac{4}{4} \leq x < 0}$$

$$\text{I} \cap \text{II} \rightarrow D_f = \left[-\frac{4}{4}, +\infty\right)$$

$$y = \frac{2x^2 + 3x + 1}{(x^2 + 5x + 1)^2} = \frac{2(1x^2 + 1.5x + 1)}{(x^2 + 1)^2} = \frac{2(2x+1)^2}{(2x+1)^4} = \frac{2}{2x+1}$$

$$2x+1 \neq 0 \quad x \neq -\frac{1}{2} \quad D_f = \mathbb{R} - \left\{-\frac{1}{2}\right\}$$

$$\frac{x^2 + 5x - 6}{x^2 - 1} = \frac{(x+6)(x-1)}{(x-1)(x+1)} = x+6 \rightarrow \boxed{C=6}$$

$$\xrightarrow{x=1} 2 = \frac{a+2}{1+b} \Rightarrow 2+2b = a+2 \rightarrow a = 2b$$

$$\xrightarrow{x=-1} -1+2 = 1 = \frac{-a+2}{-1+b} \Rightarrow -1+b = -a+2 \rightarrow a = 3-b$$

$$\begin{cases} a = 2b \\ a = 3-b \end{cases} \Rightarrow b = 1 \rightarrow \boxed{a=2} \quad \boxed{b=1} \rightarrow \boxed{a=3}$$

$$\frac{(x+1)(x-1)}{x^2 + ax + b} = \frac{x+1}{x-1}$$

$$\frac{(x+1)(x-1)}{(x-1)^2} = \frac{x+1}{x-1} \Rightarrow \begin{cases} a = -2 \\ b = 1 \\ c = 1 \end{cases}$$

$$\text{مخرج مساوی} \rightarrow D_f = D_g$$

$$\boxed{0} = a+b+c$$

$$(x-2)^2(x) + (-x-2)^2(-x)$$

$$\xrightarrow{x+1=x} \quad \xrightarrow{1-x=x} \Rightarrow (x^2 - 4x + 4)x - (x^2 + 4x + 4)x = -8x = g(x)$$

$$\left. \begin{array}{l} 1-2x = -1 \rightarrow x=1 \xrightarrow{1>0} \sqrt{1}=1 \\ 1-2x = 3 \rightarrow x=-1 \xrightarrow{-1<0} \sqrt{-1+3}=1 \end{array} \right\} f(-1)+f(3)=\boxed{2}$$

6

$$\begin{aligned} x+\sqrt{x} &= 5 \rightarrow x=4 \rightarrow \sqrt{x}+2(4)=2\sqrt{4}+12=2\sqrt{16} \\ &\downarrow \text{بالتعويض} \\ (\sqrt{x})^2 &= (5-x)^2 \\ x &= 25-10x+x^2 \\ x^2-11x+25 &= 0 \rightarrow (x-1)(x-4) \\ &\text{جواب زائد غلط} \quad \text{جواب ناقص غلط} \quad \text{جواب ناقص غلط} \quad \text{جواب ناقص غلط} \end{aligned}$$

7

$$f(x-1)+f(1-x) = (1-x)^3 + (1-x) + \underbrace{(x-1)^3 + (x-1)}_{-(1-x)^3 - (1-x)} = \boxed{0}$$

8

$$\begin{aligned} f-g &= \{(1, k)\} \xrightarrow{(1, k)} \sqrt{5-3a} + \sqrt{k-2} - \sqrt{b-4} - \sqrt{k} = k \\ &\quad \text{تفاضل جزئی} \\ 0-2-0 &= 2k \rightarrow \boxed{k=-1} \\ \left. \begin{array}{l} b-4x \geq 0 \rightarrow \frac{b}{4} \geq x \\ 4x-3a \geq 0 \rightarrow x \geq \frac{3a}{4} \end{array} \right\} x=1 = \frac{b}{4} = \frac{3a}{4} \\ &\rightarrow \boxed{b=4}, \boxed{a=\frac{4}{3}} \end{aligned} \quad \left. \begin{array}{l} \sqrt{5-\frac{b}{4}} - k = 5-2+1 \\ = 3 \end{array} \right\}$$

9

$$\begin{aligned} D_{f \cap g} &= D_f \cap D_g = [-1, 3] \\ D_g &= x \geq -1 \\ D_f &= m \geq x \\ &\Rightarrow \boxed{m=3} \\ f-g(x) &= \sqrt{x-3} + n - \sqrt{x+2} = x+n - \sqrt{x} = 5\sqrt{x} \\ &\Rightarrow \boxed{n=15\sqrt{x}-2} \end{aligned}$$

10