

$$f(n) = \begin{cases} 2n-1 & [n] > \varepsilon \\ \varepsilon n + 2 & [n] \leq \varepsilon \end{cases}$$

(1)

(2)

$$[n] > \varepsilon \Leftrightarrow n > \omega \quad \text{مثلاً}$$

$$[n] \leq \varepsilon \Leftrightarrow n < \omega \quad \text{مثلاً}$$

$$y = \sqrt{f(n) - n + 1}$$

$$\Rightarrow f(n) - n + 1 \geq 0 \quad \text{مثلاً} \quad n > \omega \Rightarrow 2n-1 - n + 1 = n \geq 0$$

$$\text{مثلاً} \Rightarrow \varepsilon n + 2 - n + 1 = 2n + \varepsilon \geq 0 \rightarrow n \geq \frac{-\varepsilon}{2}$$

$$\Rightarrow \frac{-\varepsilon}{2} \leq n < \omega \Rightarrow D = \left[\frac{-\varepsilon}{2}, +\infty \right)$$

$$2\varepsilon n^2 + 2n + 1 = 2(n+1)^2$$

$$(\varepsilon n^2 + \varepsilon n + 1) = (n+1)^2$$

$$\Rightarrow \frac{2(n+1)^2}{(n+1)^2} = \frac{2}{n+1}$$

(H)

(2)

$$n+1 \neq 0 \rightarrow n + \frac{1}{2} \Rightarrow D_f = \mathbb{R} - \left\{ \frac{1}{2} \right\}$$

$$f(n) = \begin{cases} \frac{n^2 + 2n - n - 2}{n^2 - 1} & n \neq \pm 1 \\ \frac{0n + 2}{n - 1} & n = \pm 1 \end{cases}$$

$$g(n) = n + c$$

$$f(n) = g(n) \Rightarrow n^2 + \varepsilon n^2 - n - 2 = n^2(n+c) - (n+c)$$

$$\Rightarrow (n^2 - 1)(n+c)$$

$$\Rightarrow \frac{(n^2 - 1)(n+c)}{n^2 - 1} = n + c, n \neq \pm 1 \Rightarrow n + 2 = n + c$$

$$\rightarrow n=1 \rightarrow f(1) = g(1) \rightarrow \frac{2+2}{1+2} = 2$$

$$\rightarrow n=-1 \rightarrow \frac{-1+2}{-1+2}$$

$$\Rightarrow c=2$$

$$b = \frac{1}{c} \Rightarrow b + c = \frac{5}{2}$$

$$f(u) = \frac{u-1}{u-c} \quad , \quad g(u) = \frac{u^c-1}{u^c+au+b}$$

(۲) (۵)

$$(u-1)(u+1) = (u-1)(u-c) \rightarrow$$

$$u^c+au+b = (u-1)(u-c) \rightarrow u^c - (c+1)u + c$$

$$\Rightarrow a = -c-1, b = c \rightarrow \begin{cases} \text{وقتی } c \text{ به بیاری} \\ \text{باید دانستی هم برابر} \\ \text{هستند} \end{cases} \rightarrow \begin{cases} g \rightarrow u=1 \\ \text{نقره کنی} \\ g \downarrow \\ \text{زیر یک طرح } u=1 \end{cases}$$

$$f \rightarrow u=c \rightarrow 0 \rightarrow c$$

$$\Rightarrow a = -c-1 = -1, b = 1 \Rightarrow a+b+c = -1+1+1 = 1$$

$$f(u) = (u^2)^2(u-1) \quad , \quad g(u) = f(u+1) + f(u-1) \quad (۵)$$

صفحه آخر...

$$\rightarrow f(u+1) = ((u+1)^2)^2((u+1)-1) = (u^2+2u+1)^2(u) = (u^2+2u+1)^2(u)$$

$$\Rightarrow g(u) = (u^2+2u+1)^2(u) + (u^2-2u+1)^2(u) = (u^2+2u+1)^2(u) + (u^2-2u+1)^2(u)$$

$$\Rightarrow g(u) = 2(u^2+1)(u^4-2u^2+1) = 2u^6 - 2u^4 + 2u^2 + 2u^4 - 2u^2 + 2 = 2u^6 + 2$$

$$f(1-2u) = \begin{cases} \sqrt[3]{u}+2 & u \leq 0 \\ \sqrt[3]{u} & u > 0 \end{cases} \quad (۲) (۶)$$

$$f(-1) = 1-2u = -1 \rightarrow -2u = -2 \rightarrow u = 1, f(-1) = \sqrt[3]{1} = 1$$

$$f(1/2) = 1-2u = 1/2 \rightarrow -2u = -1/2 \rightarrow u = 1/4, f(1/2) = \sqrt[3]{1/4} + 2 = 1$$

$$\Rightarrow f(-1) + f(1/2) = 1 + 1 = 2$$

(۲) (۷)

$$f(u + \sqrt{u}) = u^{\frac{1}{2}} + u^{\frac{3}{2}}, \quad u + \sqrt{u} = 4 \rightarrow \text{if } \boxed{\sqrt{u} = t}$$

$$\Rightarrow u = t^2 \rightarrow t^2 + t = 4 \rightarrow t^2 + t - 4 = 0 \rightarrow (t + \frac{1}{2})^2 = \frac{17}{4} \rightarrow t = -\frac{1}{2} \pm \frac{\sqrt{17}}{2}$$

$$\Rightarrow t = -\frac{1}{2} + \frac{\sqrt{17}}{2} \rightarrow \sqrt{u} = -\frac{1}{2} + \frac{\sqrt{17}}{2} \quad \checkmark$$

$$\Rightarrow \sqrt{u} = 2 \rightarrow u = 4 \Rightarrow \boxed{f(4) = 4^{\frac{1}{2}} + 4^{\frac{3}{2}} = 2 + 8 = 10} \quad \checkmark$$

$$f(u) = u^{\frac{1}{2}} + u^{\frac{3}{2}}, \quad \text{if } u = t^2, \quad 1 - u = -t^2$$

(۱) (۲)

$$\Rightarrow f(t^2) + f(-t^2) = \underbrace{(t^2)^{\frac{1}{2}} + (t^2)^{\frac{3}{2}}}_{f(u)} + \underbrace{(-t^2)^{\frac{1}{2}} + (-t^2)^{\frac{3}{2}}}_{f(-u)}$$

$$f(u) \leftarrow \quad \quad \quad \rightarrow f(-u)$$

$$\Rightarrow \boxed{f(u) + f(-u) = u^{\frac{1}{2}} + u^{\frac{3}{2}} - u^{\frac{1}{2}} - u^{\frac{3}{2}} = 0} \quad \checkmark$$

بیرون از رادیکال

$$f(m) = \sqrt{\epsilon m - 3a + k} - 1 \quad \text{و} \quad g(m) = \sqrt{b - \epsilon m + 3k} \quad (9)$$

صفحه آخر...

$$\{f-g\}(1/k) \rightarrow \text{"فقط } a=1 \text{ و } m \text{ متغیر است"}$$

$$1) f \rightarrow \epsilon m - 3a + k \geq 0 \Rightarrow \epsilon m \geq 3a - k \Rightarrow m \geq \frac{3a - k}{\epsilon}$$

$$2) g \rightarrow b - \epsilon m + 3k \geq 0 \Rightarrow \epsilon m \leq b + 3k$$

$$b - \epsilon m + 3k \geq 0 \Rightarrow m \leq \frac{b + 3k}{\epsilon}$$

$$\Rightarrow \frac{3a - k}{\epsilon} = 1 \quad \text{و} \quad \frac{b + 3k}{\epsilon} = 1$$

$$\Rightarrow 3a = k + \epsilon \quad \text{و} \quad b = \epsilon - 3k \quad \left\{ \begin{array}{l} f-g(1) = k \\ f-g(1) = k \end{array} \right.$$

$$\Rightarrow \epsilon - 3a + k = 0$$

$$\Rightarrow f(1) = \sqrt{0} - 1 = -1 \quad \text{و} \quad b + 3k - \epsilon = 0 \Rightarrow g(1) = \sqrt{0 + 3k + 3k}$$

$$\Rightarrow 2(-1) - 3k = k$$

$$g(m) = \sqrt{b - \epsilon m} + 3k \Rightarrow g(1) = \sqrt{b - \epsilon} + 3k$$

$$2(-1) - (\sqrt{-3k} + 3k) = k \rightarrow \sqrt{-3k} = -2 - 2k$$

$$\Rightarrow k = \frac{-(19 + \sqrt{105})}{4}$$

$$\Rightarrow 3a = k + \epsilon \quad \text{و} \quad b = \epsilon - 3k \Rightarrow 3a - \frac{b}{\epsilon} - k = (k + \epsilon) - \frac{\epsilon - 3k}{\epsilon} - k$$

$$\Rightarrow k + \epsilon - 2 + \frac{3k}{\epsilon} - k = \frac{2 + 3k}{\epsilon} \xrightarrow{\text{ضرب در } k} \frac{\sqrt{1 - 3\sqrt{105}}}{4\epsilon}$$

جواب



$$f(u) = \sqrt{m-u} + n \quad \text{و} \quad g(u) = \sqrt{2u+2}$$

$$\Rightarrow D_{f-g} = [-1, 5] \quad \text{و} \quad (f-g)(x) = 4\sqrt{2}$$

$$\rightarrow D_f \Rightarrow m-u \geq 0 \Rightarrow \boxed{u \leq m} \Rightarrow D_f = (-\infty, m]$$

$$\rightarrow D_g \Rightarrow 2u+2 \geq 0 \Rightarrow u \geq -1 \Rightarrow D_g = [-1, +\infty)$$

$$\Rightarrow D_{f-g} = D_f \cap D_g = [-1, m]$$

$$\Rightarrow [-1, m] = [-1, 5] \Rightarrow m = 5$$

$$\Rightarrow (f-g)(x) = 4\sqrt{2} \Rightarrow f(x) - g(x) = 4\sqrt{2}$$

$$\Rightarrow g(x) = \sqrt{2(x)+2} = \sqrt{10} = 3\sqrt{2}$$

$$\Rightarrow f(x) - 3\sqrt{2} = 4\sqrt{2} \Rightarrow f(x) = 7\sqrt{2}$$

$$\Rightarrow f(x) = \sqrt{u-x} + n = 4 + n = 7\sqrt{2} \Rightarrow n = 7\sqrt{2}$$

$$\Rightarrow m, n = 7 + 7\sqrt{2} - 2 = \boxed{5 + 7\sqrt{2}}$$

$$f(1-n) = (1-n-2)^2(1-n-1) = -n(n+2)^2$$

$$f(n+1) = (n+1-2)^2(n+1-1) = n(n-2)^2 \quad \text{من زوج}$$

$$g(n) = n(n-2)^2 - n(n+2)^2 = n((n-2)^2 - (n+2)^2) =$$

$$= n((n-2-n-2)(n-2+n+2)) = n(-4n) = -4n^2$$

در واقع

$$f(1-n) = -n(n-2)^2 = -n(n+2)^2 \quad \text{چون توان ۲ دارد میسر رو میخورد}$$

۹. اشتراک دامنه f و g فقط ۱ است پس:

$$D_f = \left[\frac{r_a}{r}, +\infty\right) \quad D_g = \left(-\infty, \frac{b}{r}\right] \xrightarrow{\Delta} \{1\}$$

$$\frac{r_a}{r} = \frac{b}{r} = 1 \rightarrow a = \frac{r}{r} \quad b = r$$

$$rf(1) - g(1) = K \rightarrow 2K - 2 - 3K = K \rightarrow K = -1$$

$$ra - \frac{b}{r} = K = 3$$