

$$f(n) = \begin{cases} 2n-1 & [n] \geq \varepsilon \\ \varepsilon n + 2 & [n] < \varepsilon \end{cases}$$

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$$[n] \geq \varepsilon \Leftrightarrow n \geq \varepsilon \quad \text{مثلاً}$$

$$[n] < \varepsilon \Leftrightarrow n < \varepsilon \quad \text{مثلاً}$$

$$y = \sqrt{f(n) - n + 1}$$

$$\Rightarrow f(n) - n + 1 \geq 0 \quad \text{مثلاً} \quad n \geq \varepsilon \Rightarrow 2n-1 - n + 1 = n \geq 0$$

$$\text{مثلاً} \Rightarrow \varepsilon n + 2 - n + 1 = 2n + \varepsilon \geq 0 \rightarrow n \geq \frac{-\varepsilon}{2}$$

$$\Rightarrow \frac{-\varepsilon}{2} \leq n < \varepsilon \Rightarrow D = \left[ \frac{-\varepsilon}{2}, \varepsilon + \infty \right)$$

$$2\varepsilon n^2 + 2n + 1 = \varepsilon (2n+1)^2$$

$$(\varepsilon n^2 + \varepsilon n + 1) = (\varepsilon n + 1)^2$$

$$\Rightarrow \frac{\varepsilon (2n+1)^2}{(\varepsilon n + 1)^2} = \frac{2}{1}$$

$$9 \quad 2n+1 \neq 0 \rightarrow \left[ n \neq -\frac{1}{2} \right] \Rightarrow D_f = \mathbb{R} - \left\{ -\frac{1}{2} \right\}$$

$$f(n) = \begin{cases} \frac{n^2 + 2n - n - 2}{n^2 - 1} & n \neq \pm 1 \\ \frac{an + b}{n + 1} & n = \pm 1 \end{cases}$$

$$g(n) = n + c$$

$$f(n) = g(n) \Rightarrow n^2 + \varepsilon n^2 - n - 2 = n^2 (n+c) - (n+c)$$

$$\Rightarrow (n^2 - 1)(n+c)$$

$$\Rightarrow \frac{(n^2 - 1)(n+c)}{n^2 - 1} = n + c, \quad n \neq \pm 1 \Rightarrow n + 2 = n + c$$

$$\rightarrow n=1 \rightarrow f(1) = g(1) \rightarrow \frac{1+9}{1+b} = 2$$

$$\Rightarrow c=2 \quad \text{مثلاً}$$

$$\rightarrow n=-1 \rightarrow \frac{-a+2}{-1+b}$$

$$\Rightarrow \left[ b = \frac{1}{c} \right] \quad \text{مثلاً} \quad b+c = \frac{1}{c}$$

$$f(n) = \frac{n-1}{n-c} \quad , \quad g(n) = \frac{n^c-1}{n^c+an+b}$$

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$$(n-1)(n+1) = (n-1)(n-c) \rightarrow$$

$$n^2+an+b = (n-1)(n-c) \rightarrow n^2-(c+1)n+c$$

$$\Rightarrow a = -c-1, b = c \rightarrow \begin{cases} \text{وقتی } c \text{ به بیاری} \\ \text{باید دانستیم هم برابر} \\ \text{هستند} \end{cases} \rightarrow \begin{cases} g \rightarrow n=1 \\ \text{نقره سنگی} \\ g \downarrow \\ \text{زیر یک سطح } n-1 \end{cases}$$

$$f \rightarrow n=c \rightarrow 0 \rightarrow c=1$$

$$\Rightarrow a = -c-1 = -2, b = 1 \Rightarrow a+b+c = -2+1+1 = 0$$

← جواب

$$f(n) = (n-2)^2(n-1) \quad , \quad g(n) = f(n+1) + f(n-1) \quad (5)$$

$$\rightarrow f(n+1) = ((n+1)-2)^2((n+1)-1) = (n-1)^2(n-2)$$

$$\Rightarrow g(n) = (n-2) \left[ \underbrace{n(n-2)}_{n^2-2n} + \underbrace{(n-1)^2}_{n^2-2n+1} \right] = (n-2)(2n^2-4n+1)$$

$$\Rightarrow g(n) = 2(n-2)(n^2-2n+1) = 2n^3-4n^2+2n-4n^2+8n-2 = 2n^3-8n^2+10n-2$$

$$f(1-2n) = \begin{cases} \sqrt[3]{n+2} & n \leq 0 \\ \varepsilon \sqrt{n} & n > 0 \end{cases} \quad (6)$$

$$f(-1) = 1-2n = -1 \rightarrow -2n = -2 \rightarrow n = 1, f(-1) = \varepsilon \sqrt{1} = \varepsilon$$

$$f(+2) = 1-2n = 2 \rightarrow -2n = 1 \rightarrow n = -1, f(+2) = \sqrt[3]{-1} + 2 = 1$$

$$\Rightarrow f(-1) + f(+2) = \varepsilon + 1 = 0$$

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$$f(u + \sqrt{u}) = u^{\frac{1}{2}} + u^{\frac{3}{2}}, \quad u + \sqrt{u} = 4 \rightarrow \text{if } \boxed{\sqrt{u} = t}$$

$$\Rightarrow u = t^2 \rightarrow t^2 + t = 4 \rightarrow t^2 + t - 4 = 0 \rightarrow (t + \frac{1}{2})^2 = \frac{17}{4} \rightarrow t = -\frac{1}{2} \pm \frac{\sqrt{17}}{2}$$

$$\Rightarrow t = -\frac{1}{2} + \frac{\sqrt{17}}{2} \rightarrow \sqrt{u} = -\frac{1}{2} + \frac{\sqrt{17}}{2} \quad \checkmark$$

$$\Rightarrow \sqrt{u} = 2 \rightarrow u = 4 \Rightarrow \boxed{f(4) = 4^{\frac{1}{2}} + 4^{\frac{3}{2}} = 2 + 8 = 10}$$

$$f(u) = u^{\frac{1}{2}} + u^{\frac{3}{2}}, \quad \text{if } u = 1 \Rightarrow 1 - u = -1$$

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$$\Rightarrow f(1) + f(-1) = \boxed{1^{\frac{1}{2}} + 1^{\frac{3}{2}} + (-1^{\frac{1}{2}} - 1^{\frac{3}{2}})}$$

$$\underbrace{f(1)}_{f(u)} \quad \underbrace{f(-1)}_{f(-u)}$$

$$\Rightarrow \boxed{f(u) + f(-u) = u^{\frac{1}{2}} + u^{\frac{3}{2}} - u^{\frac{1}{2}} - u^{\frac{3}{2}} = 0}$$

$$f(m) = \sqrt{\epsilon m - 3a + k} - 1 \quad \text{و} \quad g(m) = \sqrt{b - \epsilon m + 3k} \quad (9)$$

$$\{f-g\}(1/k) \rightarrow \text{"فقط } u=1 \text{ است، پس } f-g \text{ برابر است با}$$

$$1) f \rightarrow \epsilon m - 3a + k \geq 0 \Rightarrow \epsilon m \geq 3a - k \Rightarrow m \geq \frac{3a - k}{\epsilon}$$

$$2) g \rightarrow b - \epsilon m + 3k \geq 0 \Rightarrow \epsilon m \leq b + 3k$$

$$b - \epsilon m + 3k \geq 0 \Rightarrow m \leq \frac{b + 3k}{\epsilon}$$

$$\Rightarrow \frac{3a - k}{\epsilon} = 1 \quad \text{و} \quad \frac{b + 3k}{\epsilon} = 1$$

$$\Rightarrow 3a = k + \epsilon \quad \text{و} \quad b = \epsilon - 3k \quad \left\{ \begin{array}{l} f-g(1) = k \\ f-g(1) = k \end{array} \right.$$

$$\Rightarrow \epsilon - 3a + k = 0$$

$$\Rightarrow f(1) = \sqrt{0} - 1 = -1 \quad \text{و} \quad b + 3k - \epsilon = 0 \Rightarrow g(1) = \sqrt{0 + 3k - \epsilon} = \sqrt{3k - \epsilon}$$

$$\Rightarrow 2(-1) - 3k = k \quad \text{و} \quad \sqrt{3k - \epsilon} = -2 - 3k$$

$$g(m) = \sqrt{b - \epsilon m} + 3k \Rightarrow g(1) = \sqrt{b - \epsilon} + 3k$$

$$2(-1) - (\sqrt{-3k} + 3k) = k \rightarrow \sqrt{-3k} = -2 - 3k$$

$$\Rightarrow k = \frac{-19 + \sqrt{105}}{4}$$

$$\Rightarrow 3a = k + \epsilon \quad \text{و} \quad b = \epsilon - 3k \Rightarrow 3a - \frac{b}{\epsilon} - k = (k + \epsilon) - \frac{\epsilon - 3k}{\epsilon} - k$$

$$\Rightarrow k + \epsilon - 2 + \frac{3k}{\epsilon} - k = \frac{2 + 3k}{\epsilon} \quad \text{و} \quad \frac{\sqrt{1 - 3\sqrt{105}}}{4\epsilon}$$

جواب



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$$f(u) = \sqrt{m-u} + n \quad \text{و} \quad g(u) = \sqrt{2u+2}$$

$$\Rightarrow D_{f-g} = [-1, 5] \quad \text{و} \quad (f-g)(x) = 4\sqrt{2}$$

$$\rightarrow D_f \Rightarrow m-u \geq 0 \Rightarrow \boxed{u \leq m} \Rightarrow D_f = (-\infty, m]$$

$$\rightarrow D_g \Rightarrow 2u+2 \geq 0 \Rightarrow u \geq -1 \Rightarrow D_g = [-1, +\infty)$$

$$\Rightarrow D_{f-g} = D_f \cap D_g = [-1, m]$$

$$\Rightarrow [-1, m] = [-1, 5] \Rightarrow m = 5$$

$$\Rightarrow (f-g)(x) = 4\sqrt{2} \Rightarrow f(x) - g(x) = 4\sqrt{2}$$

$$\Rightarrow g(x) = \sqrt{2(x)+2} = \sqrt{8} = 2\sqrt{2}$$

$$\Rightarrow f(x) - 2\sqrt{2} = 4\sqrt{2} \Rightarrow f(x) = 6\sqrt{2}$$

$$\Rightarrow f(x) = \sqrt{5-x} + n = 6\sqrt{2} \Rightarrow n = 6\sqrt{2} - 2$$

$$\Rightarrow m, n = 5, 6\sqrt{2} - 2 \Rightarrow \boxed{5, 6\sqrt{2} - 2}$$