

$$\begin{aligned} n f(-n) - n f(n) &= 2|n| + 1 \rightarrow -n f(-n) + n^2 f(n) = -2n|n| - n \\ f(n) + n f(-n) &= 2|n| + 1 \\ \hline (n^2 + 1) f(n) &= 2|n| - 2n|n| - n \rightarrow f(n) = \frac{-2n|n| + 2|n| - n}{n^2 + 1} \\ \downarrow f &= \frac{-2n^2 + n + 1}{n^2 + 1} \quad n > 0 \\ f &= \frac{2n^2 - 3n + 1}{n^2 + 1} \quad n < 0 \end{aligned}$$

$$\begin{aligned} f(n) + f(0) &= n^2 - 3n + 3 \\ \downarrow n=0 \quad 2f(0) &= 3 \rightarrow f(0) = \frac{3}{2} \\ \left\{ \begin{aligned} f(n) &= n^2 - 3n + 3 \\ f(n^2) &= n^4 - 3n^2 + 3 \end{aligned} \right. \end{aligned}$$

$$\begin{aligned} y &= \frac{n^2 + n + 1}{n - 1} \rightarrow n^2 + n + 1 - yn + y = 0 \\ &\rightarrow n^2 + (1 - y)n + y + 1 = 0 \\ \downarrow n &= \frac{y - 1 \pm \sqrt{(y - 1)^2 - 4(y + 1)}}{2} \rightarrow \sqrt{y^2 - 2y - 3} \geq 0 \\ &\rightarrow (y - 3)(y + 1) \geq 0 \\ \frac{-1}{1} \leq \frac{y}{1} &\rightarrow R = [-1, 3] \cup [y, +\infty) \end{aligned}$$

$$\begin{aligned} \text{الف) } y &= 2n^2 - \omega n + 1 \quad \omega > 0 \quad R_f = \left[-\frac{\omega}{\lambda}, +\infty\right) \\ \downarrow n &= \frac{\omega}{\lambda} \rightarrow y = 2\left(\frac{\omega}{\lambda}\right)^2 - \frac{\omega}{\lambda} + 1 = -\frac{\omega}{\lambda} \end{aligned}$$

$$\text{ب) } y = \sqrt{-n^2 + 4n - 4} \rightarrow -n^2 + 4n - 4 \geq 0 \rightarrow (-2, 2] \xrightarrow{[-2, 2]} R_f = [0, 4]$$

$$\begin{aligned} \text{ج) } y &= \sqrt{n^2 - 3n^2 + 4n + 1} \rightarrow n^2 - 3n^2 + 4n + 1 \geq 0 \rightarrow (-\infty, +\infty) \xrightarrow{[-\infty, +\infty]} R_f = [0, +\infty) \\ \text{د) } y &= \lg(n^2 - 4n^2 + 5n + 4) \rightarrow n^2 - 4n^2 + 5n + 4 \geq 0 \rightarrow (-\infty, +\infty) \rightarrow R_f = (-\infty, +\infty) = \mathbb{R} \end{aligned}$$

$$\text{الف) } y = \frac{3n + 1}{2n - 4} \quad R_f = \mathbb{R} - \left\{\frac{3}{2}\right\}$$

$$\text{ب) } y = \sqrt{\frac{3n + 1}{n - 2}} \rightarrow \frac{3n + 1}{n - 2} \geq 0 \quad R_f = \mathbb{R} - \left\{\frac{3}{2}\right\} \xrightarrow{[1, 2) \cup (2, +\infty)} R_f = [0, +\infty) - \left\{\frac{3}{2}\right\}$$

$$\begin{aligned} \text{ج) } y &= \frac{n^2 + 4}{\sqrt{n^2 + 3}} = \frac{\sqrt{n^2 + 3} + 1}{\sqrt{n^2 + 3}} \quad \frac{a + \frac{1}{a}}{a} \geq \frac{a_{\min} + \frac{1}{a_{\min}}}{a_{\min}} \\ \downarrow \min \sqrt{n^2 + 3} &= \sqrt{3} \quad \frac{a + \frac{1}{a}}{a} \geq \frac{a_{\min} + \frac{1}{a_{\min}}}{a_{\min}} \\ R_f &= \left[\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty\right) \\ \downarrow R_f &= \left[\frac{4}{\sqrt{3}}, +\infty\right) \end{aligned}$$

$$\begin{aligned} \text{د) } y &= n^2 + \frac{1}{n^2 + 4} = \frac{n^2 + 4}{n^2 + 4} + \frac{1}{n^2 + 4} - \frac{4}{n^2 + 4} \\ \frac{a + \frac{1}{a}}{a} &\geq \frac{a_{\min} + \frac{1}{a_{\min}}}{a_{\min}} \quad R_f = \left[\frac{4}{3} + \frac{1}{3}, +\infty\right) \\ \downarrow R_f &= \left[\frac{5}{3}, +\infty\right) \end{aligned}$$

$$y = \frac{3n^2 + 4}{n^2 - 1} \leq n^2 \rightarrow n \geq 0 \rightarrow y = -4 \Rightarrow R_f = (-\infty, 4] \cup (3, +\infty)$$

$\frac{3n^2 + 4}{n^2 - 1} = 0$
 $\sqrt{\frac{3n^2 + 4}{n^2 - 1}}$
 $n \rightarrow \infty \rightarrow y \rightarrow 3$
 $n \rightarrow -\infty \rightarrow y \rightarrow 3$

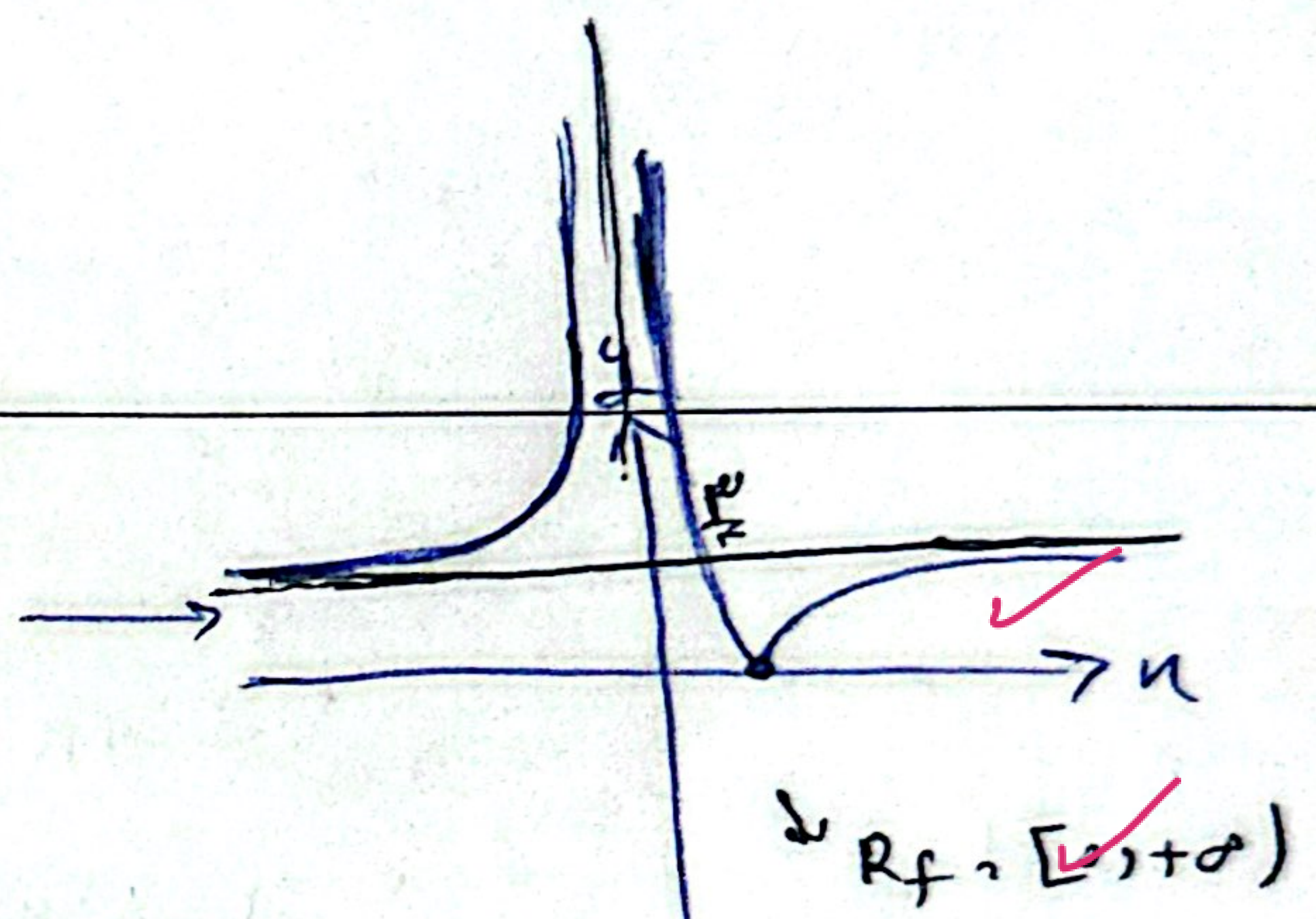
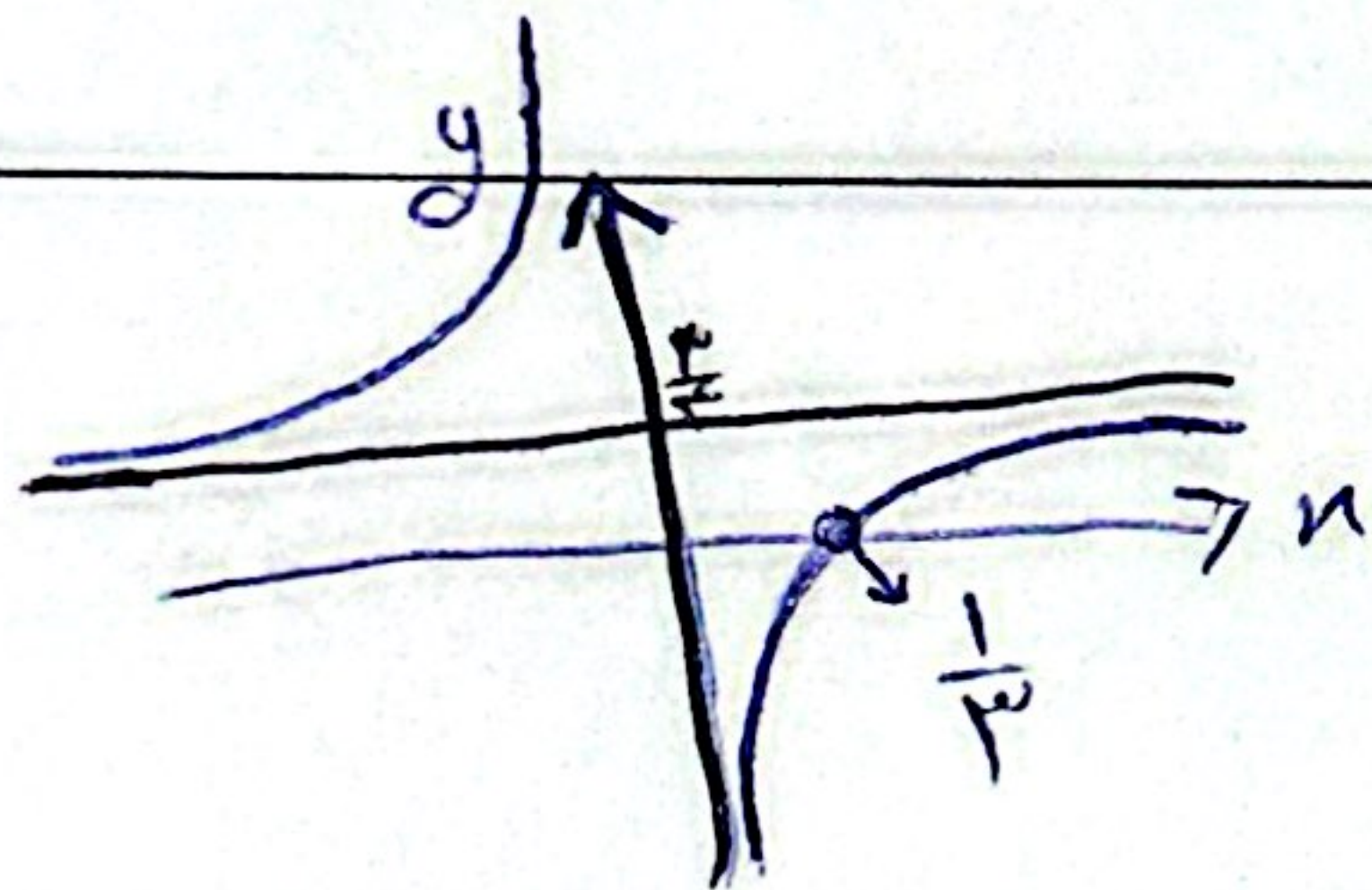
$$y = \frac{3n^2 + 4}{n^2 - 1} \rightarrow 3n^2 - yn^2 + y + 4 = 0 \rightarrow (3-y)n^2 + y + 4 = 0 \rightarrow n = \pm \frac{\sqrt{-4(y+4)(y-3)}}{4-2y}$$

$y \neq 3 \wedge (y+4)(y-3) \geq 0 \rightarrow \frac{-4y}{4-2y} \rightarrow \frac{-2y}{2-y}$

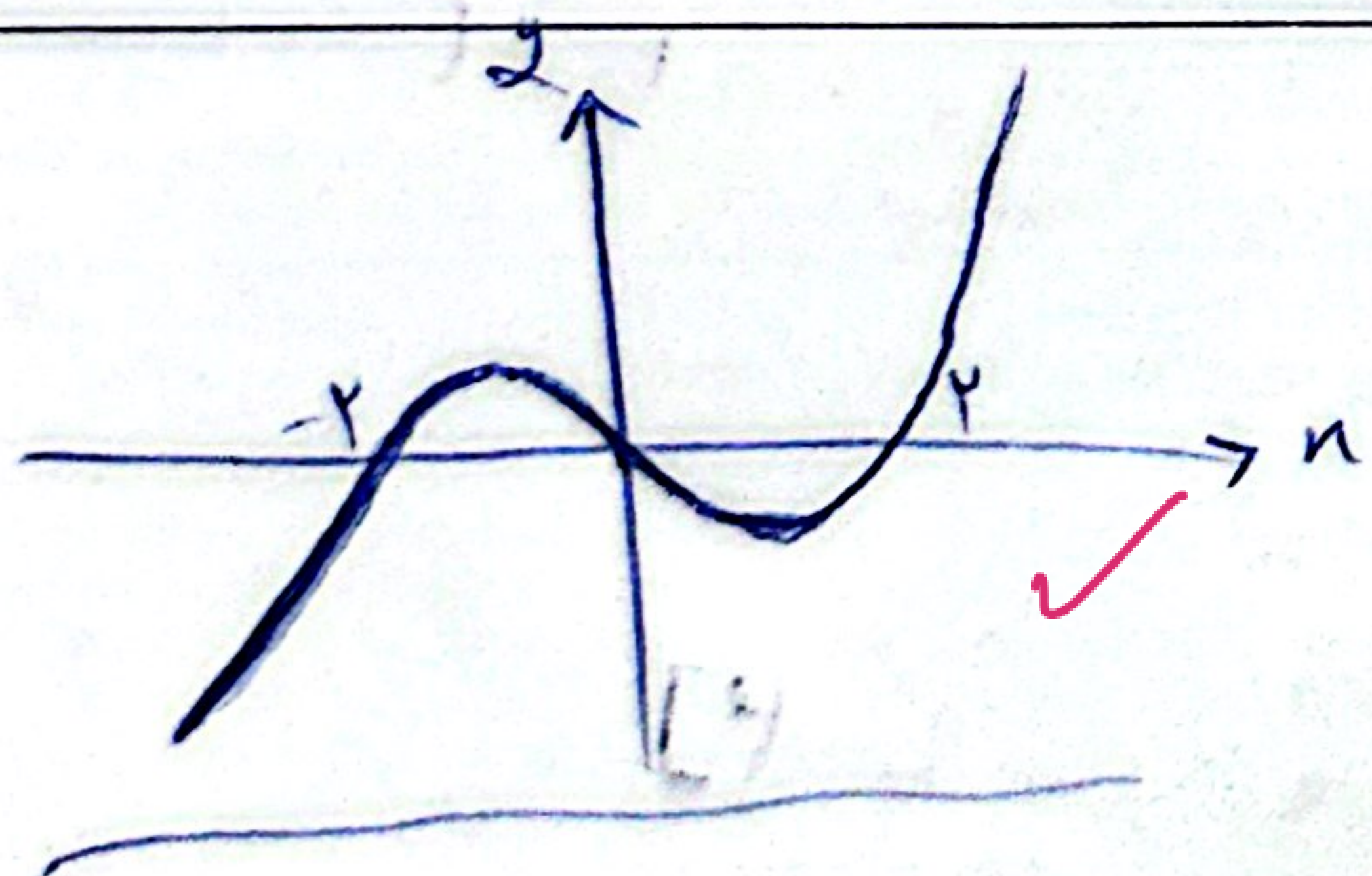
$$f(n) = \frac{4\sin n + 3}{\sin n - 3} \rightarrow \sin n = 1 \rightarrow y = -3 \quad \sin n = -1 \rightarrow y = -\frac{1}{3} \rightarrow R_f = (-\infty, -3] \cup (-\frac{1}{3}, +\infty)$$

$\sin n = 1$
 $\sin n = -1$
 $\sin n = 0$

$$الف) y = \left| \frac{3n-1}{4n} \right|$$



$$ب) y = \sqrt{\frac{9n-1}{-n+4}} = \frac{3}{4} \rightarrow R_f = \left\{ \frac{3}{4} \right\}$$



$$y = n^2 - 2n \quad n \geq 0$$

$$y = -n^2 - 2n \quad n < 0$$

$$R_f = \mathbb{R} = (-\infty, +\infty)$$

