

$$\begin{aligned} n f(-n) - n f(n) &= 2|n| + 1 \rightarrow -n f(-n) + n^2 f(n) = -2n|n| - n \\ f(n) + n f(-n) &= 2|n| + 1 \\ \hline (n^2 + 1) f(n) &= 2|n| - 2n|n| - n \rightarrow f(n) = \frac{-2n|n| + 2|n| - n}{n^2 + 1} \\ \downarrow f &= \frac{-2n^2 + n + 1}{n^2 + 1} \quad n > 0 \\ f &= \frac{2n^2 - 3n + 1}{n^2 + 1} \quad n < 0 \end{aligned}$$

$$\begin{aligned} f(n) + f(0) &= n^2 - 3n + 3 \\ \downarrow n=0 \quad 2f(0) &= 3 \rightarrow f(0) = \frac{3}{2} \\ \left\{ \begin{aligned} f(n) &= n^2 - 3n + 3 \\ f(n^2) &= n^4 - 3n^2 + 3 \end{aligned} \right. \end{aligned}$$

$$\begin{aligned} y &= \frac{n^2 + n + 1}{n - 1} \rightarrow n^2 + n + 1 - yn + y = 0 \\ &\downarrow n^2 + (1 - y)n + y + 1 = 0 \\ n &= \frac{y - 1 \pm \sqrt{(y - 1)^2 - y - 1}}{2} \xrightarrow{D_2} \sqrt{y^2 - 4y - 4} \geq 0 \\ &\downarrow (y - 2)(y + 2) \geq 0 \\ \frac{-1 - \sqrt{5}}{2} &\leq y < \frac{-1 + \sqrt{5}}{2} \rightarrow R = [-\frac{1 + \sqrt{5}}{2}, \frac{-1 + \sqrt{5}}{2}) \cup [\frac{-1 - \sqrt{5}}{2}, \frac{-1 - \sqrt{5}}{2}) \end{aligned}$$

$$\begin{aligned} \text{الف) } y &= 2n^2 - \omega n + 1 \xrightarrow{a > 0} R_f = [-\frac{\omega}{2}, +\infty) \\ \downarrow n_s &= \frac{\omega}{4} \rightarrow y_s = 2(\frac{\omega}{4})^2 - \omega(\frac{\omega}{4}) + 1 = -\frac{\omega^2}{8} \\ \text{ب) } y &= \sqrt{-n^2 + 4n - 4} \rightarrow -n^2 + 4n - 4 \xrightarrow{R} (-\infty, 4] \xrightarrow{\sqrt{(-\infty, 4]}} R_f = [0, 4] \\ \text{ج) } y &= \sqrt{n^2 - 3n^2 + 4n + 1} \rightarrow n^2 - 3n^2 + 4n + 1 \xrightarrow{R} (-\infty, +\infty) \xrightarrow{\sqrt{(-\infty, +\infty)}} R_f = [0, +\infty) \\ \text{د) } y &= \lg(n^2 - 4n^2 + 5n + 4) \rightarrow n^2 - 4n^2 + 5n + 4 \xrightarrow{R} (-\infty, +\infty) \rightarrow R_f = (-\infty, +\infty) = \mathbb{R} \end{aligned}$$

$$\begin{aligned} \text{الف) } y &= \frac{3n + 1}{2n - 4} \xrightarrow{R_f = \mathbb{R} - \{\frac{4}{3}\}} R_f = \mathbb{R} - \{\frac{4}{3}\} \\ \text{ب) } y &= \sqrt{\frac{3n + 1}{n - 2}} \xrightarrow{R_f = \mathbb{R} - \{\frac{4}{3}\}} R_f = [0, +\infty) - \{\frac{4}{3}\} \\ \text{ج) } y &= \frac{n^2 + 4}{\sqrt{n^2 + 3}} = \frac{\sqrt{n^2 + 3} + \frac{1}{\sqrt{n^2 + 3}}}{1} \xrightarrow{a + \frac{1}{a} \geq a_{\min} + \frac{1}{a_{\min}}} R_f = [\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty) \\ &\downarrow \min \sqrt{n^2 + 3} = \sqrt{3} \quad \downarrow \frac{1}{\sqrt{n^2 + 3}} \leq \frac{1}{\sqrt{3}} \\ &\downarrow a \neq 1 \quad \downarrow \frac{1}{a} \leq a \\ &\downarrow \frac{1}{a} \leq a \quad \downarrow \frac{1}{a} \leq a \\ \text{د) } y &= n^2 + \frac{1}{n^2 + 4} = \frac{n^2 + 4}{1} + \frac{1}{n^2 + 4} - 4 \xrightarrow{a + \frac{1}{a} \geq a_{\min} + \frac{1}{a_{\min}}} R_f = [4 + \frac{1}{4}, +\infty) \\ &\downarrow \frac{1}{a} \leq a \quad \downarrow \frac{1}{a} \leq a \end{aligned}$$

$$y = \frac{3n^2 + 4}{n^2 - 1} \leq n^2 \rightarrow n \geq 0 \rightarrow y \geq -4 \Rightarrow R_f = (-\infty, 4] \cup (3, +\infty)$$

$\frac{3n^2 + 4}{n^2 - 1} = 0$
 $\sqrt{\frac{3n^2 + 4}{n^2 - 1}}$
 $n \rightarrow \infty \rightarrow y \rightarrow 3$
 $n \rightarrow 0 \rightarrow y \rightarrow -4$

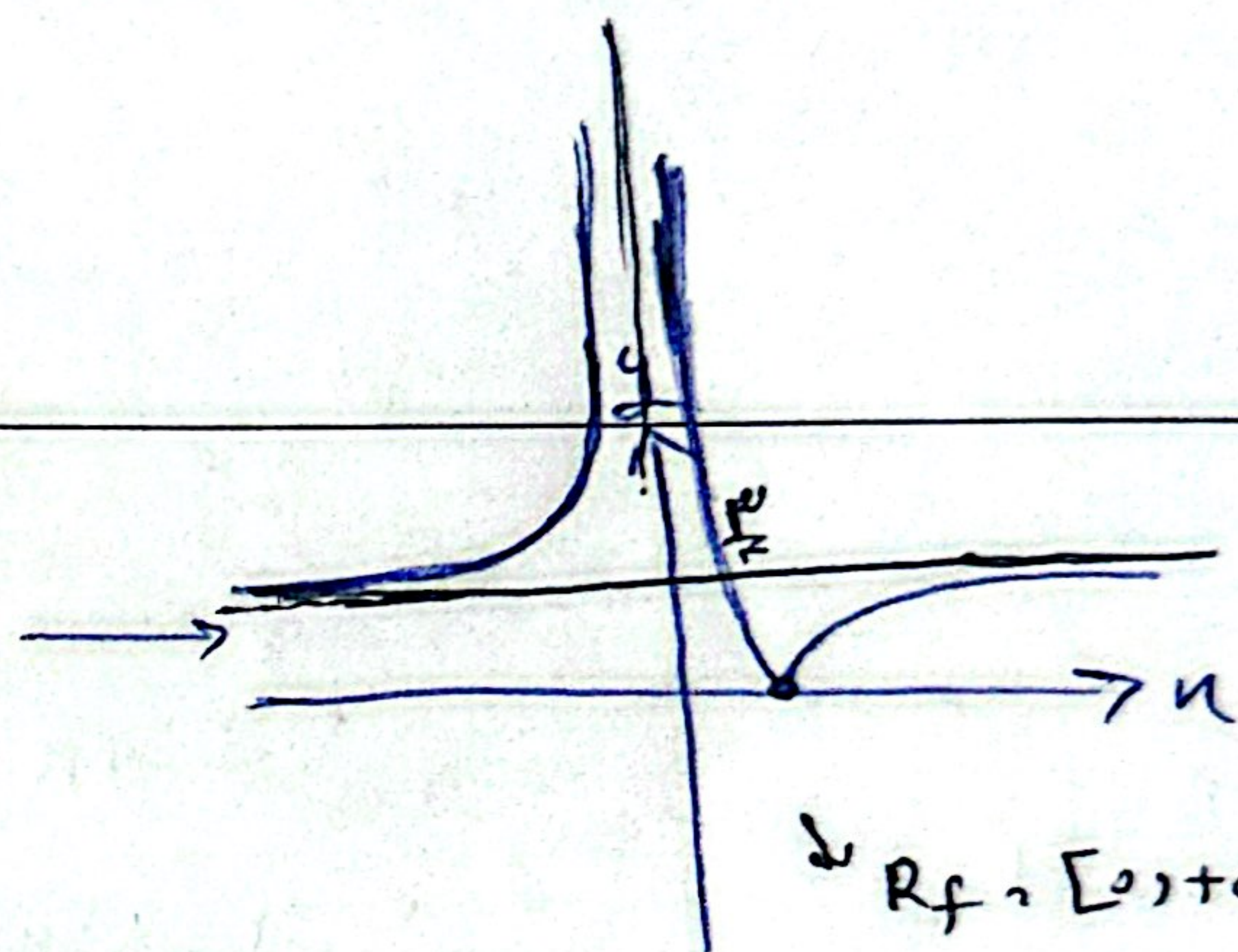
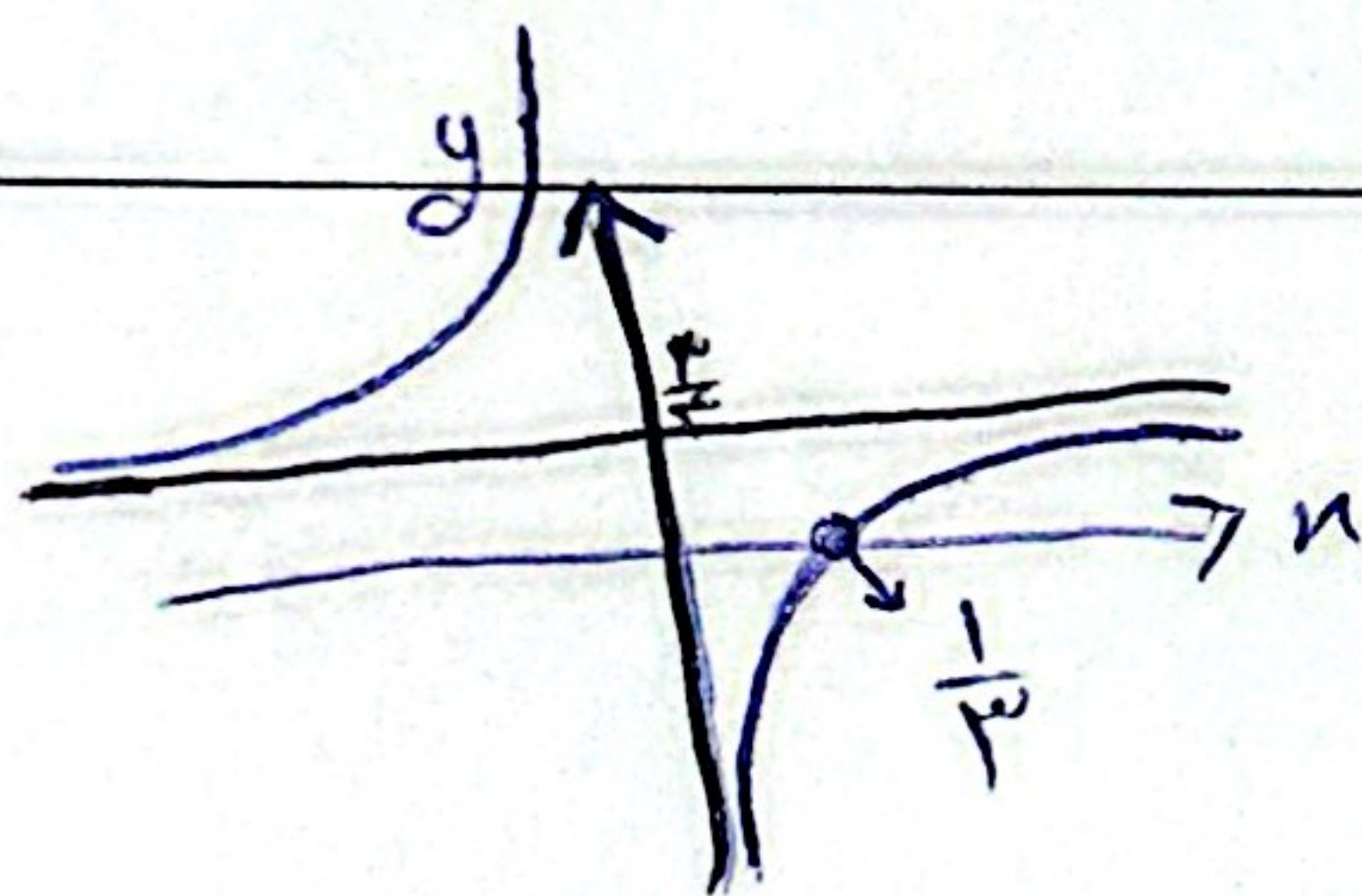
$$y = \frac{3n^2 + 4}{n^2 - 1} \rightarrow 3n^2 - yn^2 + y + 4 = 0 \rightarrow (3-y)n^2 + y + 4 = 0 \rightarrow n = \pm \frac{\sqrt{-4(y+4)(y-3)}}{4-y}$$

$y \neq 3 \wedge (y+4)(y-3) \geq 0 \rightarrow \frac{-4 \cdot 4}{4-y} \leq 0$

$$f(n) = \frac{4 \sin n + 3}{\sin n - 3} \rightarrow \sin n \geq 1 \rightarrow y \geq -3 \quad \sin n \leq -1 \rightarrow y \leq -\frac{1}{3} \Rightarrow R_f = (-\infty, -3] \cup (-\frac{1}{3}, +\infty)$$

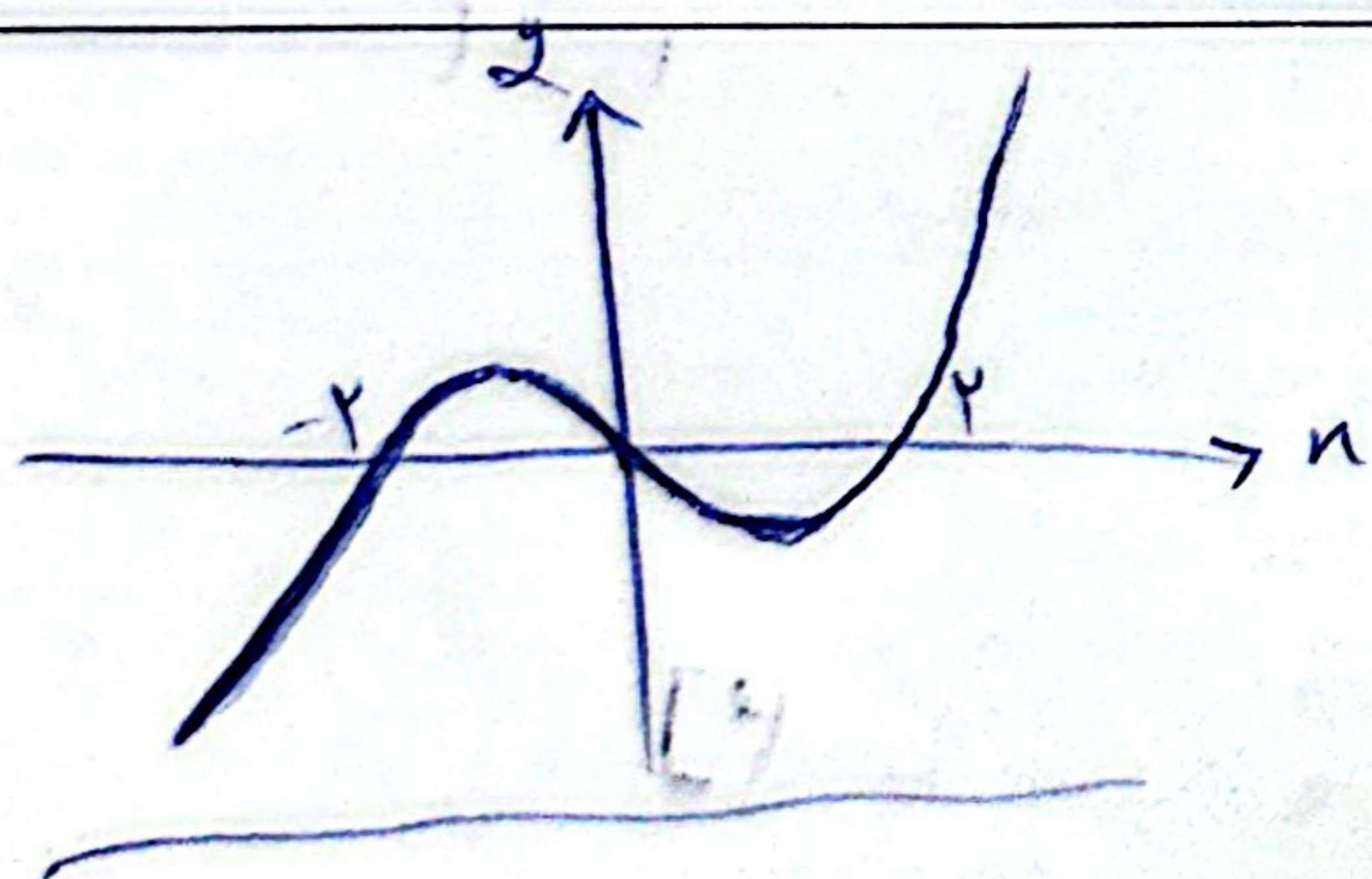
$\sin n = 3$
 $\sin n = -3$

$$\text{الف) } y = \left| \frac{3n-1}{5n} \right|$$



$$R_f = [0, +\infty)$$

$$\text{ب) } y = \sqrt{\frac{9n-1}{-n+5}} = \frac{3}{5} \rightarrow R_f = \left\{ \frac{3}{5} \right\}$$



$$y = n^2 - 2n \quad n \geq 0$$

$$y = -n^2 - 2n \quad n < 0$$

$$R_f = \mathbb{R} = (-\infty, +\infty)$$

