

$$\begin{cases} f(-n) - n f(n) = 2|n| + 1 \xrightarrow{+} 2n + 1 \\ f(n) - (-n) f(-n) = 2|-n| + 1 \rightarrow f(n) + n f(-n) = -2n + 1 \end{cases}$$

$$\begin{cases} f(-n) - f(n) = 2n + 1 \xrightarrow{(x-n)} -n f(-n) + n f(n) = -2n^2 - n \\ f(n) + n f(-n) = -2n + 1 \end{cases}$$

$$\Rightarrow n^2 f(n) + f(n) = -2n^2 - 2n + 1 \rightarrow f(n) = \frac{-2n^2 - 2n + 1}{n^2 + 1}$$

$$f(n) + f(0) = n^2 - 2n + 4 \xrightarrow{n=0} 2 f(0) = 4 \rightarrow f(0) = 2$$

$$f(n) = n^2 - 2n + 2 \Rightarrow f(n^2) = (n^2)^2 - 2(n^2) + 2 = n^4 - 2n^2 + 2$$

$$y = \frac{n^2 + n + 2}{n - 1} \rightarrow yn - y = n^2 + n + 2 \rightarrow n^2 + (1 - y)n + 2 + y = 0$$

$$\rightarrow \Delta \geq 0 \Rightarrow 1 - 2y + y^2 - 2y - 4 \geq 0 \rightarrow y^2 - 4y - 3 \geq 0$$

$$\frac{-1}{+0} \quad \frac{4}{-0} \Rightarrow R = (-\infty, -1] \cup [5, +\infty)$$

$$الف) 2n^2 - 8n + 1 \rightarrow \text{ext} \Big|_{\frac{8}{2 \times 2}} \rightarrow R = \left[-\frac{16}{2}, +\infty\right)$$

$$ب) \sqrt{-n^2 + 4n - 8} \rightarrow \text{ext} \Big|_{\frac{4}{2 \times (-1)}} \rightarrow R = [0, 2]$$

$$ج) \sqrt{n^4 - 2n^2 + 1} \rightarrow \sqrt{1R} = [0, +\infty)$$

$$د) \int (n^2 - 4n^2 + 2n + 2) \rightarrow \int 1R \rightarrow \int [0, +\infty) \rightarrow R_f = 1R$$

$$الف) y = \frac{2n + 1}{2n - 4} \rightarrow (2x - 4) \neq (2)(1) \Rightarrow R = 1R - \left\{\frac{1}{2}\right\}$$

$$ب) y = \sqrt{\frac{2n + 1}{n - 2}} \rightarrow R = \left[1R - \left\{\frac{1}{2}\right\}\right] = [0, +\infty) - \left\{\frac{1}{2}\right\}$$

$$ج) y = \frac{n^2 + 2}{\sqrt{n^2 + 1}} = \sqrt{n^2 + 1} + \frac{1}{\sqrt{n^2 + 1}} \rightarrow Ry = \left[\sqrt{1} + \frac{1}{\sqrt{1}}, +\infty\right) = \left[\frac{2\sqrt{1}}{1}, +\infty\right)$$

$$د) y = n^2 + \frac{1}{n^2 + 2} + 2 - 2 \rightarrow n^2 + 2 + \frac{1}{n^2 + 2} - 2 \rightarrow R = \left[2 + \frac{1}{2} - 2, +\infty\right) = \left[\frac{1}{2}, +\infty\right)$$

$$y = \frac{r n^r + \varepsilon}{n^r - 1} \xrightarrow{+\infty} \frac{r}{(1-\frac{1}{n})} = r \xrightarrow{\text{نصف عاقل}} R_y = (-\infty, -r] \cup (r, +\infty)$$

$$\xrightarrow{0} \frac{r}{-1} = -r$$

$$y n^r - y = r n^r + \varepsilon \rightarrow (r - y) n^r + \varepsilon + y = 0 \rightarrow \Delta \geq 0 \rightarrow r y^r + \varepsilon y - r \geq 0$$

$$\rightarrow y^r + y - r \geq 0 \rightarrow (y + \varepsilon)(y - r) \geq 0$$

$$\begin{matrix} -\varepsilon & r \\ + & - \\ 0 & 0 \end{matrix}$$

$$f(n) = \frac{r a_n + \varepsilon}{a_n - r} \xrightarrow{a_n=1} \frac{r + \varepsilon}{-r} = -r$$

$$\xrightarrow{a_n=-1} \frac{-r + \varepsilon}{-\varepsilon} = -\frac{1}{r} \rightarrow R_f = [-r, -\frac{1}{r}]$$

