

$$f(-x) - x f(x) = 2|x| + 1 \xrightarrow{x=-x} f(x) - (-x) f(-x) = 2|-x| + 1$$

$$f(x) - (-x) f(-x) = 2|x| + 1 \xrightarrow{x=-x} f(-x) + x f(x) = -2|x| + 1$$

$$f(x) = ?$$

$$\begin{cases} -x f(-x) + x f(x) = -2x - 1 \\ f(x) + x f(-x) = -2x + 1 \end{cases} \xrightarrow{+} x^2 f(x) + x f(x) = -2x^2 - 2x + 1$$

$$\rightarrow (x^2 + 1) f(x) = -2x^2 - 2x + 1 \rightarrow f(x) = \frac{-2x^2 - 2x + 1}{x^2 + 1}$$

$$f(x) + f(0) = x^2 - 3x + 4 \xrightarrow{x=0} 2f(0) = 4 \rightarrow f(0) = 2$$

$$f(x^2) = ? \rightarrow f(x) = x^2 - 3x + 4 - 2 = x^2 - 3x + 2$$

$$\rightarrow f(x^2) = (x^2)^2 - 3(x^2) + 2 = x^4 - 3x^2 + 2$$

$$y = \frac{x^2 + x + 2}{x - 1} \xrightarrow{\text{تقسیم طولی}} yx - y = x^2 + x + 2 \rightarrow x^2 + (1-y)x + 2+y = 0$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{y-1 \pm \sqrt{1+y^2-2y-4-4y}}{2} \rightarrow \Delta \geq 0$$

$$y^2 - 4y - 3 \geq 0 \rightarrow (y-7)(y+1) \geq 0$$

$$\frac{-1}{+} \frac{7}{-} \rightarrow R_f = (-\infty, -1] \cup [7, +\infty)$$

$$a) y = \frac{3x+1}{2x-4} \xrightarrow{\text{تقسیم طولی}} IR - \left\{ \frac{3}{2} \right\}$$

$$b) y = \sqrt{\frac{x+1}{x-2}} \rightarrow \sqrt{IR - \{2\}} = [0, +\infty) - \left\{ \frac{1}{2} \right\}$$

$$c) y = \frac{x^2+4}{\sqrt{x^2+3}} = \frac{x^2+3}{\sqrt{x^2+3}} + \frac{1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \xrightarrow{a+\frac{1}{a} \geq 2} \min = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$$R_f = \left[\sqrt{3} + \frac{1}{\sqrt{3}}, +\infty \right)$$

$$d) x^2 + \frac{1}{x^2+4} \rightarrow x^2 + 4 + \frac{1}{x^2+4} - 4 \rightarrow a + \frac{1}{a} - 4$$

$$e) 2x^2 - 8x + 1 \rightarrow \left(\int \right) \text{ext} \left| \frac{d}{dx} \right| \rightarrow \left(\int \right) \rightarrow R_f = \left[-\frac{16}{\lambda}, +\infty \right)$$

$$2\left(\frac{d}{2}\right)^2 - 8\left(\frac{d}{2}\right) + 1 = \frac{d^2}{2} - 4d + 1 = -\frac{d^2}{2} + \frac{1}{2} = -\frac{16}{\lambda}$$

$$f) y = \sqrt{-x^2+4x-8} \rightarrow \text{ext} \left| \frac{d}{dx} \right| \rightarrow \left(\int \right) \rightarrow R_f = [0, 2]$$

$$g) y = \sqrt{x^2 - 3x^2 + 4x + 1} \rightarrow \text{تقسیم طولی} \rightarrow \sqrt{IR} = [0, +\infty)$$

$$h) y = \log(x^2 - 4x^2 + 5x + 4) \rightarrow \log IR = \log (-\infty, +\infty) \rightarrow \log (0, +\infty)$$

$$R_f = \left[4 + \frac{1}{4} - 4, +\infty \right) = \left[\frac{1}{4}, +\infty \right)$$

$$IR \rightarrow R_f = (-\infty, +\infty)$$

$$y = \frac{3x^2 + 4}{x^2 - 1} \xrightarrow{\text{روش اقله}} y_{\min} - y = \frac{3x^2 + 4}{x^2 - 1} \rightarrow (3-y)x^2 + y + 4 = 0$$

$$x = \frac{-b \pm \sqrt{\Delta}}{2a} = \frac{0 \pm \sqrt{-4(y+4)(3-y)}}{4-2y} \rightarrow 4y^2 + 4y - 41 \geq 0$$

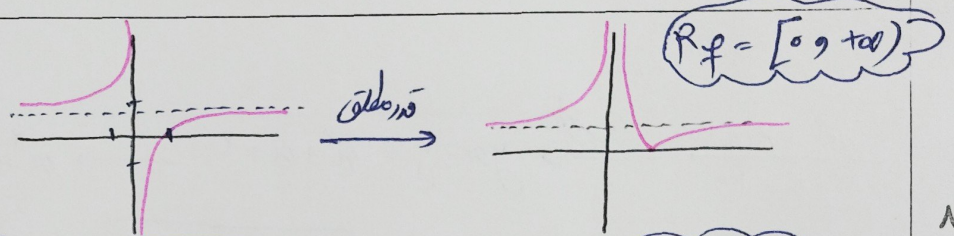
$x^2 \min \rightarrow 0 \rightarrow -4$
 $x^2 \max \rightarrow \infty \rightarrow \frac{3\infty + 4}{\infty - 1} = 3$
 $(-\infty - 4) \cup (3 + \infty)$
 بازه باز است زیرا از ۳ بزرگتر است

$$f(m) = \frac{2Sm + 4}{Sm - 3} \rightarrow \frac{a\Delta + b}{c\Delta + d}$$

$S_m = -1 \rightarrow \frac{-2+4}{-1-3} = \frac{2}{-4} = -\frac{1}{2}$
 $S_m = +1 \rightarrow \frac{2+4}{1-3} = \frac{4}{-2} = -2$

خروجی می تواند منفی شود
 بین دو بازه
 $R_f = [-2, -\frac{1}{2}]$

الف) $y = \left| \frac{3m-1}{4m} \right| = \left| \frac{3m-1}{4m+0} \right|$



ب) $y = \sqrt{\frac{9m-11}{-1+4m}} \rightarrow \sqrt{\frac{9m-11}{4m-1}} = \sqrt{\frac{9(m-2)}{4(m-2)}} = \sqrt{\frac{9}{4}} = \frac{3}{2} \rightarrow R_f = \left\{ \frac{3}{2} \right\}$
 تابع ثابت

$y = x|x| - 2x$
 $\frac{0}{-m^2 - 2m} \mid \frac{0}{m^2 - 2x}$

$D_f = \mathbb{R}$
 $R_f = \mathbb{R}$

