

$$\left. \begin{aligned} f(-x) &= x f(x) = 2|x| + 1 \\ f(x) + x f(-x) &= 2|x| + 1 \end{aligned} \right\} \times x$$

$$\Rightarrow x f(-x) - x^2 f(x) = \cancel{2|x| + 1} \quad 2|x| + 1$$

$$x f(-x) + f(x) = 2|x| + 1$$

$$\Rightarrow -x^2 f(x) - f(x) = 2|x| + 1 + x - 2|x| - 1$$

$$\Rightarrow f(x) = \frac{2|x| + 1 - 2|x| - x}{x^2 + 1}$$

$$x = 0 \Rightarrow 2f(0) = 1 \Rightarrow f(0) = \frac{1}{2}$$

$$\Rightarrow f(x) = x^2 - 2x + \frac{1}{2} \Rightarrow f(x^2) = x^4 - 2x^2 + \frac{1}{2}$$

$$x^2 + x + \frac{1}{2} = yx - y$$

$$x^2 + (1-y)x + y + \frac{1}{2} = 0$$

$$\Delta = (1-y)^2 - 4(y + \frac{1}{2}) \geq 0 \Rightarrow y^2 + 1 - 2y - 4y - 2 \geq 0$$

$$\Rightarrow y^2 - 4y - 1 \geq 0 \Rightarrow y \leq \frac{-1 - \sqrt{5}}{2} \quad \frac{-1 + \sqrt{5}}{2}$$

$$R_f = (-\infty -1] \cup [\sqrt{5} + \infty)$$

$$\text{الف) } y_{\min} = \frac{-\Delta}{4a} = \frac{-14}{4} \Rightarrow R_f = [-\frac{14}{4} + \infty)$$

$$\text{ب) } x_{\max} = \frac{1}{2} \Rightarrow y_{\max} = \frac{1}{2} \Rightarrow R_f = [0, \frac{1}{2}]$$

$$\text{ج) } \text{منطقه‌ای که برش می‌خورد} \rightarrow R \Rightarrow R_f = [0, +\infty)$$

$$\text{د) } \text{منطقه‌ای که برش نمی‌خورد} \rightarrow R \Rightarrow (0, 9) \rightarrow R_f = (-\infty, +\infty)$$

الف) $a \neq b \Rightarrow R_f = R - \left\{ \frac{a}{2} \right\} = R - \left\{ \frac{5}{2} \right\}$ -1

ب) $a \neq b \Rightarrow R_f = \sqrt{R - \{5\}} = [0, +\infty) - \{2\}$

ج) $\frac{x^2+3+1}{\sqrt{x^2+3}} = \sqrt{x^2+3} + \frac{1}{\sqrt{x^2+3}} \xrightarrow{\min} \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{4}{\sqrt{3}}$

$\Rightarrow R_f = \left[\frac{4}{\sqrt{3}}, +\infty \right)$

$\Rightarrow x^2+3 + \frac{1}{x^2+3} - 4 \xrightarrow{\min} \frac{1}{x^2+3} - 1 = \frac{1}{4} \Rightarrow R_f = \left[\frac{1}{4}, +\infty \right)$

د) $x^2+3 = yx^2-y \Rightarrow x^2(3-y) + 3+y=0 \Rightarrow (3-y)x^2+y+3=0$ -2

$\Delta \geq 0 \Rightarrow 0 - 4(3-y)(3+y) \geq 0 \Rightarrow -4(-y^2-9) \geq 0$

$\Rightarrow y^2+y-12 \geq 0 \Rightarrow \frac{-4 \pm \sqrt{16+48}}{2} \Rightarrow R_f = (-\infty, -4] \cup [3, +\infty)$

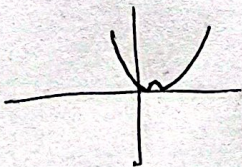
ه) $\min_{x=0} \frac{0+3}{0-1} = -3 \quad \max_{x \rightarrow +\infty} \frac{3(+\infty)}{+\infty} = 3$

منحرفه و می تواند منفی شود $\Rightarrow R_f = (-\infty, -3] \cup (3, +\infty)$

$\min \frac{-2+3}{-1-3} = \frac{-1}{-4} = \frac{1}{4} \quad \max \frac{3+3}{1-3} = -3$

منحرفه نمی تواند منفی شود $\Rightarrow R_f = \left[-3, \frac{1}{4} \right]$

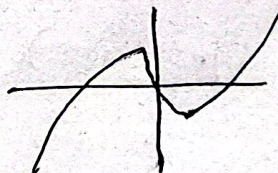
الف)



$R_f = [0, +\infty)$

ب) $\sqrt{\frac{9(x-2)}{4(x-2)}} \geq \sqrt{\frac{9}{4}} = \frac{3}{2}$

$\left. \begin{array}{l} y = x^2 - 2x \quad x \geq 0 \\ y = -x^2 - 2x \quad x < 0 \end{array} \right\}$



$R_f = R$

