

الف) $x^2 - 2x + 1 = (x-1)^2$

$$\begin{cases} f(-x) - 2f(x) = 2|x| + 1 \\ f(x) + xf(-x) = 2|x| + 1 \end{cases} \Rightarrow \begin{cases} xf(-x) - 2^x f(x) = (2|x| + 1)x \\ -f(x) - xf(-x) = -(2|x| + 1) \end{cases}$$

$$\Rightarrow (-2^x - 1)f(x) = \frac{(x-1)(2|x| + 1)}{-(x^x + 1)}$$

$$\Rightarrow f(x) = \frac{(x-1)(2|x| + 1)}{-(x^x + 1)} \checkmark$$

$$x=0 \rightarrow 2^0 f(0) = 0 - 0 + 1 = 1 \Rightarrow f(0) = 1 \Rightarrow f(x) = x^x - x^x x + 1$$

$$\Rightarrow f(2^x) = 2^x - 2^x 2^x + 1 \checkmark$$

$$yx - y = 2^x + 2x + 1 \Rightarrow x^x(1-y)x + y + 1 = 0$$

$$\Rightarrow x = \frac{y-1 \pm \sqrt{y^x - 4y + 1 - 4y - 1}}{2} = \frac{y-1 \pm \sqrt{y^x - 4y - 1}}{2} \rightarrow \frac{(y-1)(y+1)}{+1 - 1}$$

$$\Rightarrow R_y = (-\infty, -1] \cup [1, +\infty)$$

$$الف) y = 2x^x - 2x + 1 \rightarrow y_5 = 2 \times \frac{2^x}{14} - 2 \times \frac{2^x}{x} + 1 \geq -\frac{14}{x}$$

$$R_y = \left\{ -\frac{14}{x}, +\infty \right\} \checkmark$$

$$ب) y = \sqrt{-2^x + 4x - 1} \Rightarrow y = \sqrt{-2^x - 4x - 1 + 4x + 2} \Rightarrow y = \sqrt{-(x-2)^x + 4}$$

$$R_{\text{مجال}} = (-\infty, 4] \Rightarrow R_f = [0, 2] \checkmark$$

$$ج) y = \sqrt{2^x - 4x^x + 4x + 1} \Rightarrow R_{\text{مجال}} = \mathbb{R} \Rightarrow R_f = [0, +\infty) \checkmark$$

$$د) y = \log x^x - 4x^x + 4x + 1 \rightarrow 2^x - 4x^x + 4x + 1 > 0 \checkmark$$

$$\Rightarrow R_y = \mathbb{R} \checkmark$$

الف) $y = \frac{x^2+1}{x^2-4} \Rightarrow R_y = \mathbb{R} - \{\frac{4}{3}\}$

(2) - ∞

ب) $y = \sqrt{\frac{x+1}{x-2}}$ $R_{\text{معرّف}} = \mathbb{R} - \{2\} \Rightarrow R_f = [0, +\infty) - \{\frac{1}{2}\}$

ج) $y = \sqrt{x^2+4} + \frac{1}{(x^2+4)}$ $\Rightarrow y \geq \sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \Rightarrow y \geq \frac{5}{\sqrt{x^2+4}} \Rightarrow R_f = [\frac{5}{\sqrt{x^2+4}}, +\infty)$

د) $y = x^2 + 4 + \frac{1}{x^2+4} - 4 \rightarrow y \geq 4 + \frac{1}{4} - 4 \Rightarrow y \geq \frac{1}{4} \Rightarrow R_f = [\frac{1}{4}, +\infty)$

$y = \frac{x^2+4}{x^2-1} \Rightarrow yx^2 - y = x^2+4 \Rightarrow -yx^2 + yx^2 = 4+y$! $\frac{4}{x^2-1} = \frac{4+y}{x^2-1}$ - 4 (1)

$\Rightarrow x^2 = \frac{4+y}{y-4} \Rightarrow x = \pm \sqrt{\frac{4+y}{y-4}} \Rightarrow \frac{-4}{+4} = \frac{4}{-4} \Rightarrow R_f = (-\infty, 4] \cup (4, +\infty)$

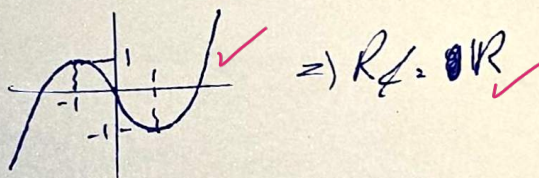
$f(x) = \frac{x \sin x + 4}{\sin x - 4}$ $\begin{cases} \frac{x}{-4} = -\frac{1}{4} \\ \frac{4}{-4} = -1 \end{cases} \Rightarrow R_f = [-1, -\frac{1}{4}]$ $\sin x \neq 0$

(2) - U

الف) $y = \left| \frac{x^2-1}{x^2} \right|$ $g(x) = \frac{x^2-1}{x^2} \Rightarrow R_g = \mathbb{R} - \{\frac{1}{x^2}\} \Rightarrow R_y = [0, +\infty) - \{\frac{1}{x^2}\}$ (1, 2)

ب) $y = \sqrt{\frac{4(x-2)}{x(x-2)}} = \frac{2}{x} = \frac{x}{x} \Rightarrow R_f = \{\frac{x}{x}\}$

$y = \begin{cases} x^2 - 2x & x \geq 0 \\ -x^2 - 2x & x < 0 \end{cases}$



$\Rightarrow R_f = \mathbb{R}$

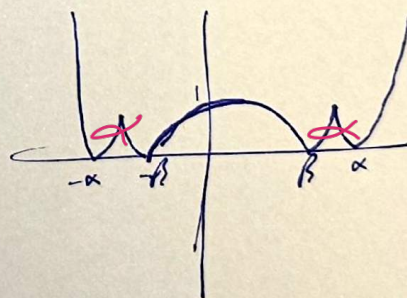
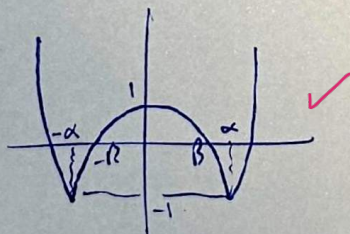
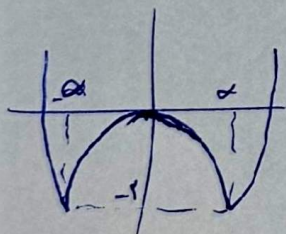
(2) - 9

$y = f(|x|)$

$g = f(|x|) + 1$

$y = |f(|x|) + 1|$

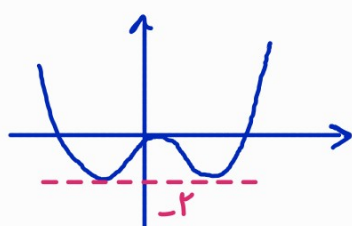
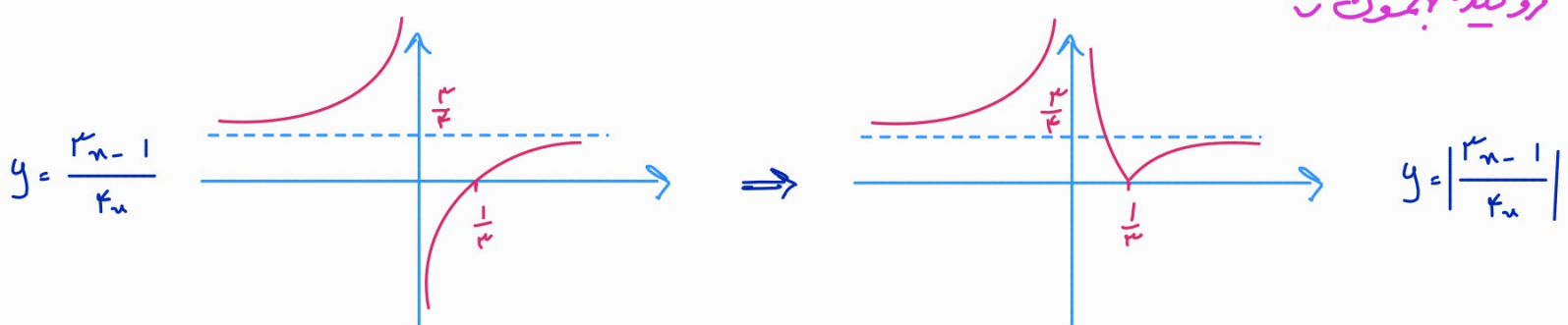
(1, 2) - 10



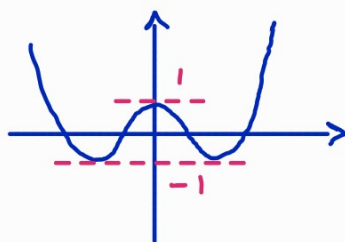
دقت!

نکته: $\square = x^2 \rightarrow \begin{cases} x^2 = 0 \rightarrow y = -4 \\ x^2 \sim \infty \rightarrow y = 3 \end{cases} \xrightarrow[\text{مغز نشه}]{\text{مخرج مینونه}} R_y = (-\infty, -4] \cup (3, +\infty)$

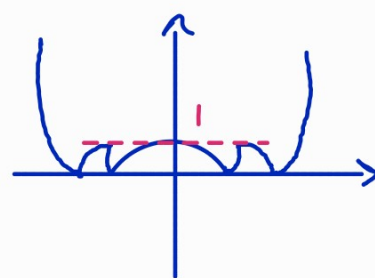
۸- اول برد عبارت داخل قدر مطلق رو حساب میکنیم بعد قدر مطلق رو بگیریم فقط خواست باشه که $y = \frac{3}{4}$ هم هست داخل برد $y = -\frac{3}{4}$ عضو برد عبارت داخل قدر مطلق هست و وقتی قدر مطلق بگیریم $y = \frac{3}{4}$ رو دیده بهموندن



$$y = f(1-x)$$



$$y = f(|x|) + 1$$



$$y = |f(|x|) + 1|$$