

$$\begin{aligned}
 f(-x) - x f(x) &= 2|x|+1 \xrightarrow{x \rightarrow -x} x f(-x) - x^2 f(x) = 2(|x|+1) \\
 f(x) + x f(-x) &= 2|-x|+1 = 2|x|+1 \\
 (1-x^2) f(x) &= (x+1)(2|x|+1) \\
 \Rightarrow f(x) &= \frac{(x+1)(2|x|+1)}{1-x^2} = \frac{2|x|+1}{1-x} \quad x \neq -1
 \end{aligned}$$

$$\begin{aligned}
 f(x) + f(x^2) &= x^2 - 2x + 4 \\
 x f(x) &= 4 \Rightarrow f(x) = \frac{4}{x} \Rightarrow f(x) = x^2 - 2x + 4 \\
 f(x^2) &\Rightarrow (x^2)^2 - 2(x^2) + 4 = x^4 - 2x^2 + 4
 \end{aligned}$$

$$\begin{aligned}
 y &= \frac{x^2 + x + 2}{x-1} = y \Rightarrow y = x^2 - 2x + 4 \\
 x^2 + (1-y)x + 2-y &\Rightarrow \Delta \geq 0 \\
 x &= \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2-y)}}{2} \sim y^2 - 2y + 1 - 8y + 4 - 8 = y^2 - 10y - 7 \\
 R_f &= (-\infty, -1] \cup [7, +\infty) \Leftarrow \frac{-1 \pm \sqrt{1+48}}{2} \cdot (y-7)(y+1) \geq 0
 \end{aligned}$$

$$\begin{aligned}
 1) y &= 2x^2 - 8x + 1 \quad 2) y = \sqrt{x^2 + 4x - 8} \quad 3) y = \sqrt{x^2 - 4x^2 + 4x + 1} \quad y = L_y(x^2 - 4x^2 + 4x + 1) \\
 x_{min} = \frac{b}{2a} &= \frac{4}{4} = 1 \quad x_{min} = \frac{-b}{2a} = \frac{-4}{2} = -2 \quad R_f = [0, +\infty) \Rightarrow R_f = \mathbb{R} \\
 y_{min} = \frac{c-b^2/4a}{a} &= \frac{1-16}{4} = -\frac{15}{4} \quad y_{min} = \frac{c-b^2/4a}{a} = \frac{-8-16}{2} = -12 \\
 \Rightarrow R_f &= [-\frac{15}{4}, +\infty) \Rightarrow R_f = [0, 4] \\
 &\cup
 \end{aligned}$$

$$\begin{aligned}
 y &= \frac{x+1}{x-4} \quad y = \sqrt{\frac{x+1}{x-4}} \quad y = \frac{x^2+1}{\sqrt{x^2+4}} \quad y = x^2 + \frac{1}{x^2+4} = \frac{x^2+1}{x^2+4} \\
 R_f &= \mathbb{R} - \left\{ \frac{4}{3} \right\} \quad R_f = [0, +\infty) - \left\{ \frac{1}{2} \right\} \quad \frac{x^2+1}{\sqrt{x^2+4}} + \frac{1}{\sqrt{x^2+4}} = \Rightarrow \left[\frac{1}{4}, +\infty \right) - \frac{1}{2} = \\
 &\mathbb{R} - \left\{ \frac{4}{3} \right\} \quad \left[-\frac{1}{2}, +\infty \right) \\
 R_f &= \left[\frac{1}{4}, +\infty \right) \quad R_f = \left[\frac{1}{4}, +\infty \right)
 \end{aligned}$$

$$y = \frac{x^2 + 1}{x^2 - 1} - 1(-y^2 - y + 1) \geq 0$$

$$x^2 + 1 - x^2 - y + y^2 + y - 1 \geq 0$$

$$(x-y)x^2 + 1 + y = 0$$

$$\Delta \geq 0 \Rightarrow b^2 - 4ac = (-1)^2 - 4(1)(1+y) \geq 0$$

$$1 - 4(1+y) \geq 0$$

$$1 - 4 - 4y \geq 0$$

$$-3 - 4y \geq 0$$

$$-4y \geq 3$$

$$y \leq -\frac{3}{4}$$

$$R_f = (-\infty, -\frac{3}{4}] \cup [\frac{3}{4}, +\infty)$$

$$f(x) = \frac{\sin x + x}{\sin x - x}$$

$$\frac{y}{-1} = -x$$

$$\frac{x}{-1} = -\frac{1}{x}$$

$$R_f = [-\frac{1}{2}, -\frac{1}{2}]$$

$$x) y = \left| \frac{x^2 - 1}{x^2} \right|$$

$$R_f = |R - \{\frac{1}{x}\}|$$

$$R_f = [0, +\infty) - \{\frac{1}{x}\}$$

$$y = x|x| - x^2$$

$$-x^2 - x^2 \mid x^2 - x^2$$

$$R_f = \mathbb{R}$$

