

$$\begin{cases} x f(-x) - x^2 f(x) = x|2x| + x \\ -f(x) - x f(-x) = -|2x| - 1 \end{cases} \rightarrow$$

$$\begin{aligned} \rightarrow -(x^2+1)f(x) &= x|2x| + x - |2x| - 1 \Rightarrow \\ \rightarrow f(x) &= \frac{x|2x| + x - |2x| - 1}{-(x^2+1)} \end{aligned}$$

$$\begin{aligned} x=0 \rightarrow f(0)+f(0) &= 4 \rightarrow f(0)=2 \Rightarrow f(x) = x^2 - cx + 2 \rightarrow \\ \rightarrow f(x^2) &= (x^2)^2 - 2(x^2) + 2 \rightarrow \boxed{f(x^2) = x^4 - 2x^2 + 2} \end{aligned}$$

$$\begin{aligned} yx - y &= x^2 + x + 2 \rightarrow x^2 + \frac{(1-y)x}{y} + \frac{2+y}{y} = 0 \rightarrow x = \frac{-b \pm \sqrt{\Delta}}{2a} \\ \rightarrow y &= 1 \pm \sqrt{y^2 - 4y - 4} \rightarrow y^2 - 4y - 4 \geq 0 \rightarrow \frac{-1}{+} \quad \frac{5}{-} \quad \frac{4}{+} \\ \Rightarrow R_{f(x)} &= (-\infty -1] \cup [5 \infty) \end{aligned}$$

$$\begin{aligned} \frac{a}{x} = x_{ex} \Rightarrow 1 = y_{ex} \text{ و } a > 0 \rightarrow R_x &= [1 \infty) \quad \text{الف} \\ \frac{-y}{-x} = x_{ex} \Rightarrow 2 = y_{ex} \text{ و } a < 0 \rightarrow R_x &= [-\infty 2] \rightarrow R_y = [0 \infty) \quad \text{ب} \\ R_{\text{برابر}} &= \mathbb{R} \rightarrow R_y = [0 \infty) \quad \text{ج} \end{aligned}$$

د- اگر آنچه که چند جمله ای با بزرگترین توان مورد نیاز به اعداد حقیقی است بین جملی $\log$ هر عددی می تواند باشد (با شرط دامنه) پس هر تابع برابری $\mathbb{R}$ است

$$\begin{aligned} x^2 y - 4y &= 2x + 1 \rightarrow x = \frac{1+2y}{y^2-4} \rightarrow y^2 - 4 \neq 0 \rightarrow y \neq \pm 2 \rightarrow R = \mathbb{R} - \{2\} \\ y^2 = \frac{2x+1}{x-2} \rightarrow x = \frac{1+2y}{y^2-4} \rightarrow x = \pm \sqrt{\frac{1+2y}{y^2-4}} + \frac{-1}{2} - \frac{2}{2} \rightarrow R = \mathbb{R} - [-2, 2] \\ \sqrt{2x^2+3} + \frac{1}{\sqrt{2x^2+3}} \rightarrow a + \frac{1}{a} \mid \min(a) = \sqrt{3} \rightarrow R = [\sqrt{3} + \frac{1}{\sqrt{3}} \infty) \\ x^2 + 2 + \frac{1}{x^2+2} - 2 \rightarrow a + \frac{1}{a} - 2 \mid \min(a) = 1 \rightarrow R = [\frac{1}{2} \infty) \\ \rightarrow R = [\frac{1}{2} + \frac{1}{2} - 2 \infty) \end{aligned}$$

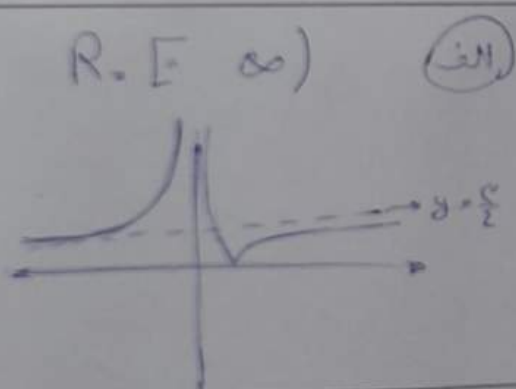
$\text{Max } \sin x = 1 \rightarrow \frac{1+\varepsilon}{1-\varepsilon} = \frac{1}{-1} = -1$
 $\text{Min } \sin x = -1 \rightarrow \frac{-1+\varepsilon}{-1-\varepsilon} = \frac{1}{-\varepsilon} = -\frac{1}{\varepsilon}$

$\rightarrow R_{f(n)} = [-1, -\frac{1}{\varepsilon}]$

$x^2 y - y = 3x^2 + \varepsilon \rightarrow x^2 = \frac{3+y}{y-3} \rightarrow x = \pm \sqrt{\frac{3+y}{y-3}}$

$\text{Max } (x') = \infty \rightarrow f(x) = \frac{3(\infty)+\varepsilon}{\infty-1} \rightarrow \varepsilon$
 $\text{Min } (x') = 0 \rightarrow f(x) = -\varepsilon$

$\Rightarrow R = [-\varepsilon, \varepsilon]$



$y = \sqrt{\frac{9(x-1)}{2(x+1)}} = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$

$R = \left\{ \pm \frac{3}{\sqrt{2}} \right\}$

$-x^2 \leq x^2 - 2x$
 $R = \mathbb{R}$

