

$$\alpha = x \rightarrow f(-x) - x f(x) = 2|x| + 1 \xrightarrow{\times x} x f(-x) - x^2 f(x) = 2x|x| + x \quad (1)$$

$$\alpha = -x \rightarrow f(x) + x f(-x) = 2|x| + 1 \quad (2)$$

$$(2) - (1) = (1+x^2) f(x) = (1-x)(2|x|+1) \rightarrow f(x) = \frac{(1-x)(2|x|+1)}{1+x^2}$$

$$f(x) + f(0) = x^2 - 3x + 4 \rightarrow f(x) = ax^2 + bx + c, f(0) = c$$

$$\rightarrow f(x) + f(0) = ax^2 + bx + 4 \rightarrow a, b = 1, c = 3 \rightarrow f(x) = x^2 - 3x + 3$$

$$\rightarrow f(x^2) = x^4 - 3x^2 + 3$$

$$y = \frac{x^2 + x + 2}{x - 1} \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1-y)x + 2+y = 0$$

$$x = \frac{y-1 \pm \sqrt{y^2 y + 1 - 4 - 4y}}{2} \rightarrow y^2 - 4y - 3 \geq 0 \rightarrow y \in (-\infty, -1] \cup [3, +\infty)$$

$$y = -x^2 + 4x - 5 \rightarrow R_f = (-\infty, -\frac{5}{1}) = (-\infty, -5] \quad (1)$$

$$\rightarrow \sqrt{R_f} = [0, 2]$$

$$y = 2x^2 - 5x + 1 \rightarrow R_f = [-\frac{5}{8}, +\infty) \quad (2)$$

$$\rightarrow R_f = [-\frac{1}{2}, +\infty)$$

$$y = \log x^2 - 4x^2 + 5x + 4 \rightarrow \mathbb{R}$$

$$y \in \mathbb{R}$$

$$y = x^2 - 3x^2 + 4x + 1 \rightarrow R_f = \mathbb{R}$$

$$\rightarrow \sqrt{R_f} = [0, +\infty)$$

$$y = \sqrt{\frac{x+1}{x-2}} \rightarrow \frac{x+1}{x-2} \in \mathbb{R} - \{f\}$$

$$\rightarrow R_f \in [0, +\infty) - \{f\}$$

$$y = \frac{3x+1}{2x-4} \rightarrow \text{معرکینک} \quad (3)$$

$$\rightarrow R_f \in \mathbb{R} - \{\frac{3}{2}\}$$

$$y = x^2 + \frac{1}{x^2 + 2} = x^2 + 2 + \frac{1}{x^2 + 2} - 2$$

(4)

$$y = \frac{x^2 + 2}{\sqrt{x^2 + 2}} = \frac{x^2 + 2 + 1}{\sqrt{x^2 + 2}} = \sqrt{x^2 + 2} + \frac{1}{\sqrt{x^2 + 2}} \quad (4)$$

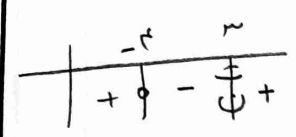
$$\rightarrow R_f \in [\frac{1}{2} - 2, +\infty) \rightarrow R_f \in [\frac{1}{2}, +\infty)$$

$$\rightarrow R_f \in [\frac{2\sqrt{2}}{3}, +\infty)$$

روش اصلی:

$$y = \frac{rx^r + r}{x^r - 1} \rightarrow x^r = \frac{y+r}{y-r}$$

$$\rightarrow \frac{y+r}{y-r} \geq 0 \rightarrow y \in (-\infty, -r] \cup (r, +\infty)$$



$$y = \frac{rx^r + r}{x^r - 1}$$

$$\left. \begin{array}{l} \max x^r = +\infty \rightarrow y = \frac{r \cdot \infty}{\infty} = r \\ \min x^r = 0 \rightarrow y = \frac{r}{-1} = -r \\ x^r - 1 = 0 \end{array} \right\} y \in (-\infty, -r] \cup (r, +\infty)$$

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$$y = \frac{r \sin x + r}{\sin x - r}$$

$$\left. \begin{array}{l} \max \sin x = 1 \rightarrow y = \frac{r}{-r} = -1 \\ \min \sin x = -1 \rightarrow y = \frac{r}{-r} = -1 \\ \sin x - r \neq 0 \end{array} \right\} y \in [-1, -1]$$

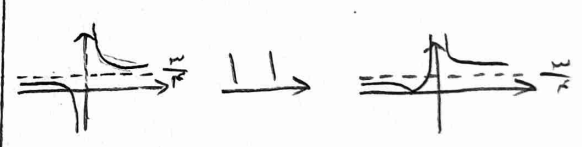
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$$y = \sqrt{\frac{g(m-r)}{f(m-r)}} = \frac{r}{r}$$

$$\rightarrow R_f \in \left\{ \frac{r}{r} \right\}$$

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$$y = \left| \frac{rx-1}{rx} \right|$$

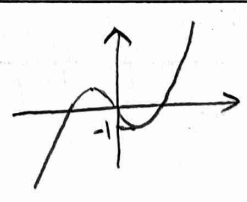


$$\rightarrow R_f \in [0, +\infty)$$

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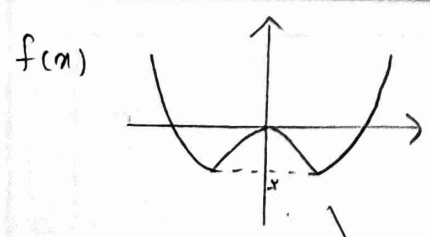
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$$f(x) = \begin{cases} x^r - rx & x \geq 0 \\ -x^r - rx & x < 0 \end{cases}$$



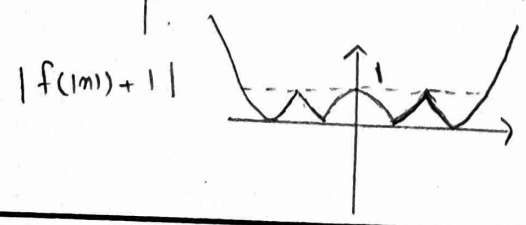
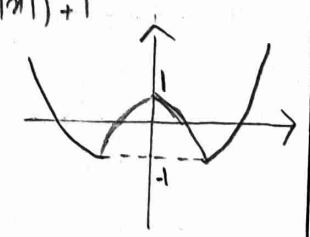
$$\rightarrow R_f \in \mathbb{R}$$

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چون نمودار $f(x)$ نسبت به محور
 $f(x) = f(|x|)$ متقارن است پس:

$$f(|x|) + 1$$



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