

$$\begin{aligned} -x &\rightarrow f(x) + x f(-x) = -2x + 1 \\ x &\rightarrow f(-x) - x f(x) = 2x + 1 \end{aligned} \quad \left\{ \begin{aligned} + &\rightarrow (1-x) f(x) + (1+x) f(-x) = 1 \\ - &\rightarrow (1+x) f(x) + (x-1) f(-x) = -2x \end{aligned} \right.$$

$$x f(-x) - x^2 f(x) = x(2x+1)$$

$$f(x) + x(f(-x)) = 2|x| + 1 = 2|x| + 1$$

$$\rightarrow x^2 f(x) - f(x) = 2x|x| + x - 2|x| - 1$$

$$\rightarrow -f(x)(x^2+1) = 2|x|(x-1) + x - 1 \rightarrow f(x) = \frac{-(x-1)/(2|x|+1)}{x^2+1}$$

$$x=0 \rightarrow 2f(0) = 1 \rightarrow f(0) = \frac{1}{2}$$

$$x=x^2 \rightarrow f(x^2) + \frac{1}{2} = x^2 - \frac{1}{2}x^2 + \frac{1}{2} \rightarrow f(x^2) = x^2 - \frac{1}{2}x^2 + \frac{1}{2}$$

$$\frac{x^2 + x + 1}{x-1} = \frac{x^2 - x + 2(x-1)}{x-1} = x+2 \quad y = x+2 \xrightarrow{f} y = x-2$$

$$\boxed{R \setminus R}$$

$$y = \frac{x^2 + x + 1}{x-1} \rightarrow x^2 + (1-y)x + y = 0$$

$$\rightarrow R_y = (-\infty, -1] \cup [y, +\infty)$$

$$x = \frac{y-1 \pm \sqrt{y^2 - 4y - 4}}{2} \Delta \geq 0 \rightarrow y^2 - 4y - 4 \geq 0 \quad \frac{-1}{2} \pm \frac{\sqrt{17}}{2}$$

$$الف) y = 2(x - \frac{\sqrt{17}}{2})^2 - \frac{17}{4} + 1 \rightarrow f' = \frac{1}{\sqrt{17}}(x + \frac{\sqrt{17}}{2}) + \frac{1}{2} \rightarrow D_{f'} = R_{f'} = x + \frac{\sqrt{17}}{2} > \frac{\sqrt{17}}{2}$$

$$ب) \frac{-1}{f_x} = \frac{-(2x-1)}{-1} = f \quad R_f = [0, 1]$$

$$ج) R = R^+ \cup \{0\} = [0, +\infty)$$

$$د) R = R^+ = (0, +\infty)$$

$$الف) 2xy - 2y = 2x + 1 \rightarrow x = \frac{y+1}{2y-2} \rightarrow y = \frac{y+1}{2x-2} \quad R_y = D_{y'} = R - \{\frac{1}{2}\}$$

$$ب) y^2 = \frac{x+1}{x-2} \rightarrow xy^2 - 2y^2 = x+1 \rightarrow x = \frac{y^2+1}{y^2-2} \rightarrow y = \frac{y^2+1}{x^2-2} \quad R_y = D_{y'} = R^+ - \{\pm\sqrt{2}\}$$

$$ج) = \sqrt{x^2+1} + \frac{1}{\sqrt{x^2+1}} \quad \min = \sqrt{2} \rightarrow R = [R - (-\sqrt{2}, \sqrt{2})] \cup R - [\frac{\sqrt{2}}{2}, +\infty) = (0, 2) \cup (2, +\infty)$$

$$د) = x^2 + \frac{1}{x^2+2} - \frac{1}{2} \rightarrow R = (-\infty, -1] \cup [1, +\infty)$$

$$y^2 - y = 3x^2 + 4 \rightarrow x^2 = \frac{y+4}{y-3} \rightarrow y = \sqrt{\frac{y+4}{y-3}} \quad \frac{-2}{\pm 2} = \frac{2}{\pm 2}$$

$$D_f = R_f = [-\infty, 3] \cup (3, +\infty)$$

$$x=0 \rightarrow y=-4$$

$$x=+\infty \rightarrow y=3$$

$$R = R - (-4, 3) = (-\infty, -4] \cup [3, +\infty)$$

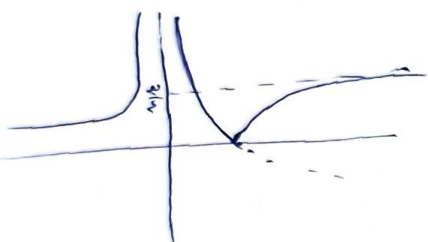
دو سر x^2 و 0 می تواند شود

$$f(x) - y = \frac{y \sin x + 4}{\sin x - 3} \rightarrow y \sin x - 3y = y \sin x + 4 \rightarrow \sin x = \frac{y+4}{y-3}$$

$$-1 \leq \frac{y+4}{y-3} \leq 1 \rightarrow \frac{y+4}{y-3} = 1$$

$$0 \leq \frac{y+4}{y-3} + 1 \rightarrow \frac{y+4}{y-3} \rightarrow \frac{-\frac{1}{3} + 2}{+1 - 1 +} \rightarrow R - (-\frac{1}{3}, 2]$$

$$\frac{y+4}{y-3} - 1 \leq 0 \rightarrow \frac{y+4}{y-3} \leq 0 \rightarrow \frac{-3}{+1 - 1 +} \rightarrow [-3, 2]$$

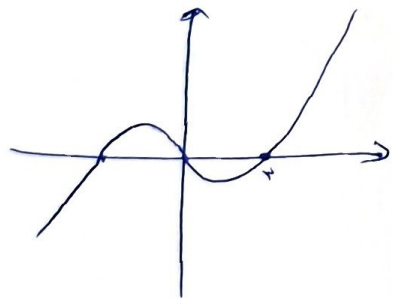


$$R = [0, +\infty)$$

$$y^2 = 9x - 18 \rightarrow x = \frac{y^2 + 18}{9}$$

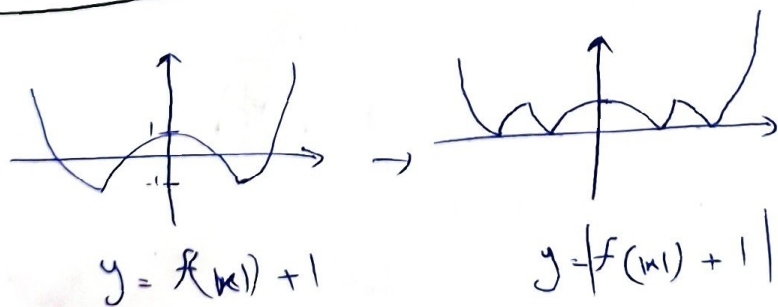
$$x = \pm \frac{y^2 + 18}{9} \rightarrow y^2 - 9 = 0 \Rightarrow y = \sqrt{\frac{9}{2}} = \frac{3}{\sqrt{2}}$$

$$R = R - \left\{ \pm \frac{3}{\sqrt{2}} \right\}$$



$$x < 0 \rightarrow y = -x^2 - 2x = -(x^2 + 2x)$$

$$R_y = R$$



$$y = |f(x) + 1|$$