

$x = -x \rightarrow f(x) + x f(-x) = 2 x + 1$ $x = x \rightarrow (f(-x) - x f(x) = 2 x + 1) \cdot (-x)$ $-x f(-x) + x^2 f(x) = 2x x + x$ $\Rightarrow (x^2 + 1) f(x) = 2 x + 1 + 2x x + x$ $\rightarrow f(x) = \begin{cases} \frac{x^2 + 3x + 1}{x^2 + 1} & x \geq 0 \\ \frac{-x^2 + 3x + 1}{x^2 + 1} & x < 0 \end{cases}$	۱
$x = 0 \rightarrow 2 f(0) = 1 \rightarrow f(0) = \frac{1}{2}$ $x = x^2 \rightarrow f(x^2) + \frac{1}{2} = x^4 - \frac{1}{2} x^2 + \frac{1}{2} \rightarrow f(x^2) = x^4 - \frac{1}{2} x^2 + \frac{1}{2} = (x^2 - \frac{1}{4})^2 - \frac{1}{4}$	۲
$x^2 + x + 2 \rightarrow yx - y = x^2 + x + 2 \rightarrow x^2 + (1-y)x + 2 + y = 0 \xrightarrow{\Delta} \frac{-(1-y) \pm \sqrt{(1-y)^2 - 4(2+y)}}{2}$ $\rightarrow (1-y)^2 - 4(2+y) \geq 0 \rightarrow 1 - 2y + y^2 - 8 - 4y = y^2 - 6y - 7 \geq 0$ $(y-7)(y+1) \geq 0$ $\frac{-1}{+\infty} \frac{7}{-\infty} \Rightarrow R_f = \mathbb{R} - (-1, 7)$	۳
الف) $a > 0 \rightarrow \text{min}$ $\frac{-b}{2a} = \frac{5}{4} \quad \frac{-\Delta}{4a} = \frac{-18+1}{4} = -\frac{17}{4}$ دامنه $= (\frac{5}{4}, -\frac{17}{4})$ $R_f = [-\frac{17}{4}, +\infty)$ ب) $a < 0$ دامنه $= (3, 4)$ $R_f = \sqrt{(-\infty, 4)} = [0, 2]$ ج) $a > 0 \rightarrow R = \mathbb{R}$ $R_f = \sqrt{\mathbb{R}} = [0, +\infty)$ د) $a < 0 \rightarrow R = \mathbb{R}$ $\log^{(-\infty, +\infty)} \rightarrow \mathbb{R} = R_f$	۴
الف) $R_f = \mathbb{R} - \{\frac{1}{2}\}$ ب) $R_f = \sqrt{\mathbb{R} - \{4\}} = [0, +\infty) - \{2\}$ ج) $\sqrt{x^2 + 3} + \frac{1}{\sqrt{x^2 + 3}} \xrightarrow{\sqrt{x^2 + 3} = a > 0} R_f = [t, +\infty)$ $\rightarrow t = a + \frac{1}{a}$ جایگاه اولی است و مقدار قبل از آن $x^2 = 0 \rightarrow \sqrt{3} + \frac{1}{\sqrt{3}} = \frac{3+1}{\sqrt{3}} = \frac{4\sqrt{3}}{3} \Rightarrow R_f = [\frac{4\sqrt{3}}{3}, +\infty)$ د) $x^2 + 4 + \frac{1}{x^2 + 4} - 4$ $\left. \begin{aligned} & a + \frac{1}{a}, a > 0 \\ & x^2 = 0 \text{ کمترین مقدار} \\ & \text{مقدار تابع} = 4 + \frac{1}{4} = \frac{17}{4} \end{aligned} \right\} R_f = [\frac{17}{4}, +\infty)$	۵

$$y = x^2 - y = x^2 + 4 \rightarrow y = x^2 + 4$$

$$\rightarrow (x-y)x^2 + 4 + y = 0 \rightarrow \frac{\sqrt{-4(x-y)(4+y)}}{x(x-y)} \geq 0$$

$$x-y \neq 0 \rightarrow \frac{-4}{x(x-y)} \rightarrow R_f = \mathbb{R} - (-4, 3]$$

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$$\square \text{ روش } \rightarrow \frac{x^2 + 4}{x^2 - 1} \rightarrow \left\{ \begin{array}{l} x^2 = +\infty \rightarrow \frac{x^2}{x^2} = 1 \text{ باز} \\ x^2 = 0 \rightarrow -4 \text{ بسته} \end{array} \right\} R_f = (-\infty, -4] \cup (3, +\infty) = \mathbb{R} - (-4, 3]$$

بین دوری

روش ۲

$$\frac{x \sin x + 4}{\sin x - 3} \rightarrow \left\{ \begin{array}{l} \sin x = 1 \rightarrow \frac{4}{-2} = -2 \\ \sin x = -1 \rightarrow \frac{4}{-4} = -1 \end{array} \right\} R_f = [-2, -\frac{1}{3}]$$

$\sin x \neq 3$
 $-1 \leq \sin x \leq 1$
 داخل محدوده

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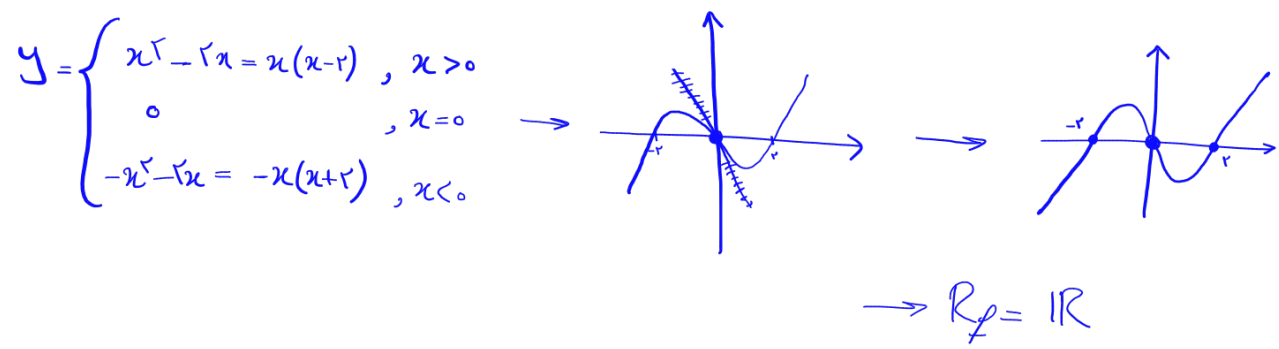
$$\text{الف) } y = \frac{x^2 - 1}{x^2} \rightarrow R_f = \mathbb{R} - \{\frac{1}{x}\}$$

$$\text{ب) } y = \sqrt{\frac{9(x-1)}{x(x-1)}} = \frac{3}{x} \rightarrow R_f = \{\frac{3}{x}\}$$

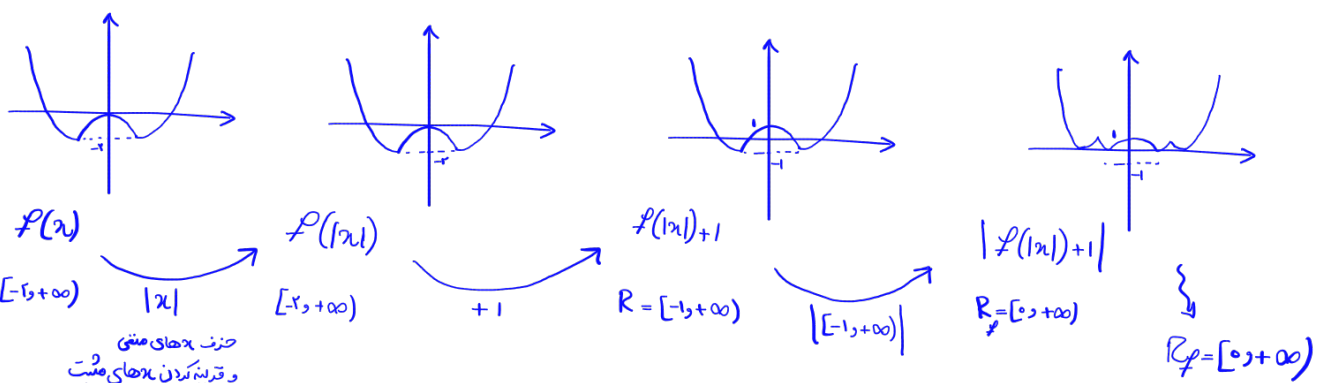
$$y = \left| \frac{x^2 - 1}{x^2} \right| \rightarrow R_f = |\mathbb{R} - \{\frac{1}{x}\}| = [0, +\infty) - \{\frac{1}{x}\}$$

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$$\square \rightarrow \left\{ \begin{array}{l} +\infty \rightarrow \frac{x^2}{x^2} = 1 \text{ باز} \\ -\infty \rightarrow \frac{-x^2}{-x^2} = 1 \text{ باز} \end{array} \right\} \xrightarrow{\frac{1}{x} \neq 0} R_f = |\mathbb{R} - \{\frac{1}{x}\}| = [0, +\infty) - \{\frac{1}{x}\}$$



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