

$x=0 \rightarrow f(0)=1$
 $x=1 \rightarrow f(-1)-f(1)=3 \Rightarrow f(1)=3$
 $x=-1 \rightarrow f(1)+f(-1)=3 \Rightarrow f(1)=0$
 $ax^2+bx+c \Rightarrow \begin{cases} x=0 \rightarrow c=1 \\ x=1 \rightarrow a+b+1=0 \Rightarrow a+b=-1 \\ x=-1 \rightarrow a-b+1=3 \Rightarrow a-b=2 \end{cases} \Rightarrow a=\frac{1}{2}, b=-\frac{3}{2}$
 $\Rightarrow f(x)=\frac{1}{2}x^2-\frac{3}{2}x+1$
 صفحه بعدی را بنویسیم
 چرته؟

$f(0)=4 \rightarrow f(0)=3 \Rightarrow f(x^2)=x^2-3x^2+3$

$y = \frac{x^2+x+2}{x-1} \Rightarrow yx-y=x^2+x+2 \Rightarrow x^2+x(1-y)+y+2=0 \Rightarrow x = \frac{y-1 \pm \sqrt{y^2-y+1-y-1}}{2}$
 $\Rightarrow \Delta \geq 0 \rightarrow y^2-2y-1 \geq 0 \Rightarrow (y-1)^2 \geq 2 \Rightarrow y-1 \geq \sqrt{2} \Rightarrow y \geq 1+\sqrt{2}$

$\sqrt{-x^2+4x-0}$
 $\begin{cases} -\frac{b}{2a} = 2 \\ y = 0 \end{cases} \Rightarrow [-\infty, 4] \rightarrow [0, 4]$
 $0f=R \Rightarrow Rf=R$
 $2x^2-0x+1 \rightarrow$
 $\begin{cases} -\frac{b}{2a} = 0 \\ y = \frac{1}{2x} \end{cases} \Rightarrow Rf = [-\frac{1}{2}, +\infty)$
 $R = (-\infty, +\infty) \rightarrow [0, +\infty)$

$y = \frac{ax+b}{cx+d} \Rightarrow Rf = R - \left\{ \frac{a}{c} \right\}$
 $\Rightarrow Rf = R - \left\{ \frac{2}{1} \right\}$
 $\sqrt{0} \Rightarrow Rf = R - \{0\} \Rightarrow Rf = [0, +\infty)$
 $x^2+4 + \frac{1}{x^2+4} - 4 = a + \frac{1}{a} - 4$
 $\Rightarrow a + \frac{1}{a} \geq 4 \Rightarrow [4, +\infty)$
 $\sqrt{x^2+4} + \frac{1}{\sqrt{x^2+4}} \geq a + \frac{1}{a} \Rightarrow Rf \geq 2$
 $\sqrt{4} + \frac{1}{\sqrt{4}} = 2 \Rightarrow \sqrt{4} + \frac{1}{\sqrt{4}} = 2 \Rightarrow Rf = [2, +\infty)$

$y = \frac{3x^2+2}{x^2-1}$
 $\begin{cases} \text{max } y = 3 \Rightarrow Rf = (-\infty, -1] \cup (3, +\infty) \\ \text{min } y = -1 \Rightarrow Rf = (-\infty, -1] \cup (3, +\infty) \end{cases}$

$$f(n) = \frac{r \sum_{k=1}^n k + r}{\sum_{k=1}^n k - r} \quad \begin{matrix} \text{max} \\ \text{min} \end{matrix} \quad \begin{matrix} \frac{r}{-r} = -1 \\ \frac{r}{-0} = -\frac{r}{\epsilon_0} \end{matrix} \quad \text{و نیز } R_f = [-1, -\frac{1}{r}] \quad (2) \quad (V)$$

$$(الف) \quad y = \left| \frac{r^n - 1}{\epsilon n} \right| \xrightarrow{\text{محدود}} -\frac{1}{n} + \frac{r}{\epsilon} \quad \begin{matrix} \text{graph 1} \\ \text{graph 2} \end{matrix} \quad (2) \quad (A)$$

$\Rightarrow R = [0, +\infty)$

$$(ب) \quad y = \sqrt{\frac{9n - 1}{\epsilon n - 1}} \quad \Rightarrow R_f = \left[\frac{3}{r}, \infty \right) \quad (2) \quad (A)$$

$$y = x|x| - rn \quad \begin{cases} n^r - rn & n > 0 \\ 0 & n = 0 \\ -n^r - rn & n < 0 \end{cases} \quad \begin{matrix} \text{graph} \\ \Rightarrow R_f = \mathbb{R} \end{matrix} \quad (2) \quad (A)$$

$$f(n) = \begin{matrix} \text{graph 1} \\ \text{graph 2} \end{matrix} \quad \xrightarrow{f(n)} \begin{matrix} \text{graph 3} \\ \text{graph 4} \end{matrix} \quad \begin{matrix} +1 \\ \text{graph 5} \end{matrix} \quad \xrightarrow{| \cdot |} \begin{matrix} \text{graph 6} \end{matrix} \quad (2) \quad (A)$$

۱- جای n و $-n$ قرار می دهیم و یک دستگاه معادلات می بنویسیم:

$$\begin{aligned} f(-n) - n f(n) &= r|n| + 1 \quad \xrightarrow{\times n} \quad \begin{cases} n f(-n) - n^2 f(n) = r|n| + n & (I) \\ f(n) + n f(-n) = r|n| + 1 & (II) \end{cases} \\ f(n) + n f(-n) &= r|n| + 1 \end{aligned}$$

$$\underline{II - I} \rightarrow (1 + n^2) f(n) = r(1 - n)|n| + 1 - n \quad \Rightarrow f(n) = \frac{(1 - n)(r|n| + 1)}{1 + n^2}$$