

$$x=0 \rightarrow f(0)=1$$

$$\begin{aligned} x=1 &\rightarrow f(-1)-f(1)=2 \\ x=-1 &\rightarrow f(1)+f(-1)=2 \end{aligned} \Rightarrow \begin{cases} f(-1)=2 \\ f(1)=0 \end{cases}$$

$$ax^2+bx+c \Rightarrow \begin{cases} x=0 \rightarrow c=1 \\ x=1 \rightarrow a+b+1=0 \Rightarrow a+b=-1 \\ x=-1 \rightarrow a-b+1=2 \Rightarrow a-b=1 \end{cases} \Rightarrow a=\frac{1}{2}, b=-\frac{3}{2}$$

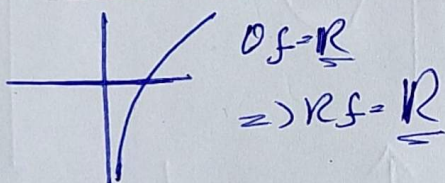
$$\Rightarrow f(x) = \frac{1}{2}x^2 - \frac{3}{2}x + 1$$

$$f(x) = 4 \rightarrow f(x)=2 \Rightarrow f(x^2) = x^2 - 3x^2 + 2$$

$$y = \frac{x^2+x+2}{x-1} \Rightarrow yx-y = x^2+x+2 \Rightarrow x^2+x(1-y)+y+2=0 \Rightarrow x = \frac{y-1 \pm \sqrt{y^2-y+1-y-1}}{2}$$

$$\Rightarrow \Delta \geq 0 \rightarrow y^2-2y-1 \geq 0 \Rightarrow (y-1)^2 \geq 2 \Rightarrow y-1 \geq \sqrt{2} \text{ or } y-1 \leq -\sqrt{2} \Rightarrow y \geq 1+\sqrt{2} \text{ or } y \leq 1-\sqrt{2}$$

$$\sqrt{-x^2+4x-0} \Rightarrow \begin{cases} -\frac{b}{2a} = 2 \\ y = 0 \end{cases} \Rightarrow [-\infty, 4] \xrightarrow{R_f} [0, 4]$$



$$\begin{aligned} 2x^2-0x+1 &\rightarrow \text{Graph of } y=2x^2+1 \text{ is a parabola opening upwards with vertex at } (0,1). \\ \begin{cases} -\frac{b}{2a} = 0 \\ y = 1 \end{cases} \Rightarrow R_f = [1, +\infty) \end{aligned}$$

$$\text{Since } y \geq 1, R = (-\infty, +\infty) \xrightarrow{R_f} [0, +\infty)$$

$$\sqrt{y} \geq 0 \Rightarrow R_f = \mathbb{R}_+ - \{0\} \xrightarrow{\sqrt{\cdot}} R_f = [0, +\infty) - \{0\} = (0, +\infty)$$

$$y = \frac{ax+b}{cx+d} \Rightarrow R_f = \mathbb{R} - \left\{ \frac{a}{c} \right\} \Rightarrow R_f = \mathbb{R} - \left\{ \frac{2}{1} \right\} = \mathbb{R} - \{2\}$$

$$x^2+2 + \frac{1}{x^2+2} - 2 = a + \frac{1}{a} - 2$$

$$\begin{aligned} \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}} &\geq a + \frac{1}{a} \Rightarrow R_f \geq 2 \\ \text{Since } a \geq \sqrt{2}, \sqrt{x^2+2} &\geq \sqrt{2} \Rightarrow \sqrt{x^2+2} + \frac{1}{\sqrt{x^2+2}} \geq \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}} \end{aligned}$$

$$\begin{aligned} y = \frac{3x^2+2}{x^2-1} &\begin{cases} \text{Since } y \geq 2, x^2 = -\frac{y-2}{y-3} \Rightarrow x^2 = \frac{y+2}{y-3} \Rightarrow \frac{y+2}{y-3} \geq 0 \Rightarrow y \geq 3 \text{ or } y \leq -2 \\ \Rightarrow R_f = (-\infty, -2] \cup [3, +\infty) \\ \text{Since } y \leq -2, \frac{y+2}{y-3} = 0 \Rightarrow y = -2 \Rightarrow R_f = (-\infty, -2] \cup (3, +\infty) \end{cases} \end{aligned}$$

$$f(n) = \frac{r \sin n + r}{\sin n - r}$$

$\xrightarrow{\text{max}} \frac{r}{-r} = -1$
 $\xrightarrow{\text{min}} \frac{r}{-0} = -\frac{r}{0} \rightarrow -\infty$

$\Rightarrow R_f = [-1, -\frac{1}{r}]$

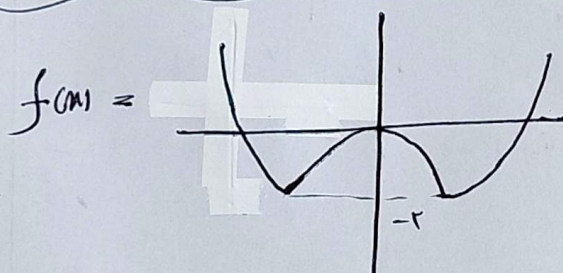
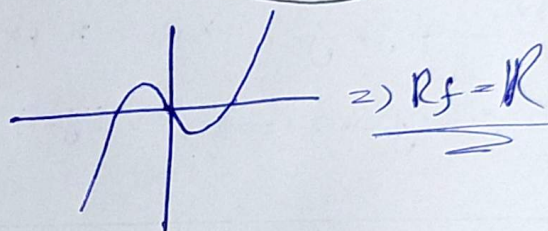
(الف) $y = \left| \frac{r n - 1}{\epsilon n} \right| \xrightarrow{\text{L'Hôpital}} -\frac{1}{n} + \frac{r}{\epsilon}$

$\Rightarrow R = [0, +\infty)$

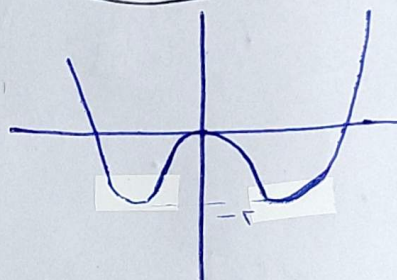
(ب) $y = \sqrt{\frac{9n-1}{\epsilon n-1}}$

$\Rightarrow R_f = \left\{ \frac{r}{r} \right\}$

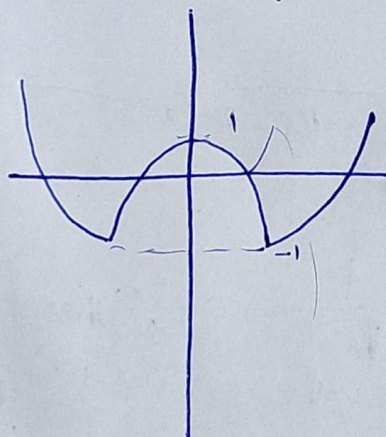
$$y = x|x| - r x \begin{cases} x^2 - r x & x > 0 \\ 0 & x = 0 \\ -x^2 - r x & x < 0 \end{cases}$$



$f(n)$



$+1$



$|0|$

