

$$f_{(-n)} - n(f_n) = 2|n| + 1 \xrightarrow{n \rightarrow \infty} n f_{(-n)} - n^2 f_n = n(2|n| + 1) \left\{ \begin{array}{l} -n^2 f_{(-n)} - f_n \\ = 2n|n| + n - 2|n| - 1 \end{array} \right. \quad (۲)$$

$$f_n + n(f_{(-n)}) = 2|-n| + 1 = 2|n| + 1$$

$$\rightarrow -f_n(n^2 + 1) = 2|n|(n-1) + (n-1) \rightarrow f_n = -\frac{(n-1)(2|n|+1)}{n^2+1} \quad \checkmark$$

$$f_n + f_0 = n^2 - 2n + 1 \rightarrow 2f_0 = 1 \rightarrow f_0 = \frac{1}{2} \quad (۲)$$

$$\rightarrow f_n = n^2 - 2n + 1 \rightarrow f_{(n^2)} = n^2 - 2n^{\frac{1}{2}} + 1 \quad \checkmark$$

$$y = \frac{n^2 + n + 2}{n-1} \rightarrow y n - y = n^2 + n + 2 \rightarrow n^2 + (1-y)n + (2+y) = 0 \quad (۲)$$

$$\rightarrow n = \frac{y+1 \pm \sqrt{(1-y)^2 - 4(2+y)}}{2} \rightarrow (1-y)^2 - 4(2+y) \geq 0 \rightarrow y^2 - 6y - 7 \geq 0$$

$$\rightarrow R_f = (-\infty, -1] \cup [7, +\infty) \quad \checkmark$$

$$الف) y = 2n^2 - 8n + 1 \xrightarrow{a > 0} \begin{cases} \frac{-b}{2a} = \frac{4}{2} \rightarrow y_{min} = \frac{-17}{2} \rightarrow R_f = [\frac{-17}{2}, +\infty) \end{cases} \quad (۲)$$

$$ب) y = \sqrt{-n^2 + 4n - 5} \xrightarrow{a < 0} y_{max} = 1 \rightarrow R_f = \sqrt{(-\infty, 1]} = [0, 1] \quad \checkmark$$

$$ج) y = \sqrt{n^2 - 2n^2 + 4n + 1} \rightarrow R_f = \sqrt{IR} = [0, +\infty) \quad \checkmark$$

$$د) y = \log(n^2 - 4n^2 + 4n + 4) \rightarrow R_{(n^2 - 4n^2 + 4n + 4)} = IR \xrightarrow{\log} R_f = (-\infty, +\infty) \quad \checkmark$$

$$الف) \frac{2n+1}{2n-7} \xrightarrow{\text{تجزیه کسری}} R_f = IR - \left\{ \frac{a}{c} \right\} = IR - \left\{ \frac{2}{7} \right\} \quad \checkmark$$

$$ب) y = \sqrt{\frac{2n+1}{n-7}} \rightarrow R_f = \sqrt{IR - 7} = [0, +\infty) - \left\{ \frac{7}{2} \right\} \quad \checkmark$$

$$ج) y = \sqrt{n^2 + 4} + \frac{1}{\sqrt{n^2 + 4}} \rightarrow y_{min} = \sqrt{4} + \frac{1}{\sqrt{4}} = \frac{5}{2} \rightarrow R_f = \left[\frac{5}{2}, +\infty \right) \quad \checkmark$$

$$د) y = (n^2 + 4 + \frac{1}{n^2 + 4}) - 4 \rightarrow R_f = \left[\frac{1}{4}, +\infty \right) \quad \checkmark$$

$$y = \frac{r n^r + \varepsilon}{n^r - 1} \rightarrow y n^r - y = r n^r + \varepsilon \rightarrow (y - r) n^r - (y + \varepsilon) = 0$$

$$\rightarrow n = \frac{\pm \sqrt{r(y+\varepsilon)(y-r)}}{r(y-r)} \rightarrow \begin{cases} y \neq r \\ \varepsilon(y+\varepsilon)(y-r) \geq 0 \rightarrow \frac{-\varepsilon}{r} \leq y \leq \frac{-\varepsilon}{r} \end{cases}$$

$$\rightarrow R_f = \mathbb{R} - (-\varepsilon, r]$$

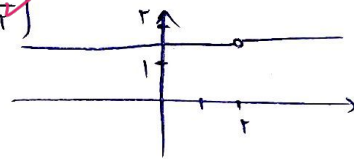
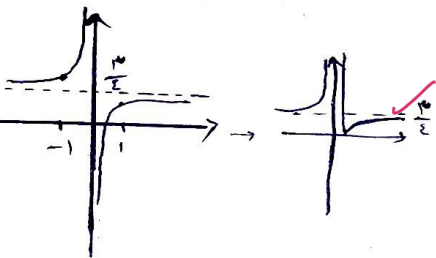
$$n^r = \square \rightarrow \begin{cases} n = 0 \rightarrow y = -\varepsilon \\ n = +\infty \rightarrow y = r \end{cases} \xrightarrow{\text{خرج/شروط}} R_f = (-\infty, -\varepsilon] \cup (r, +\infty)$$

$$f(n) = \frac{r \sin n + \varepsilon}{\sin n - r} \rightarrow \begin{cases} \sin n = 1 \rightarrow y = -r \\ \sin n = -1 \rightarrow y = -\frac{1}{r} \end{cases} \xrightarrow{\text{رسم دایره}} R_f = [-r, -\frac{1}{r}]$$

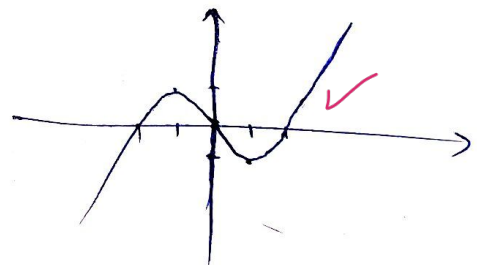
$$y = \left| \frac{r n - 1}{\varepsilon n} \right| \rightarrow R_f = [0, +\infty)$$

$$y = \sqrt{\frac{r n - 1}{\varepsilon n - 1}} = \sqrt{\frac{r}{\varepsilon}} \quad n \neq 1$$

$$\rightarrow R_f = \left\{ \frac{r}{\varepsilon} \right\}$$



$$y = n|n| - r n \rightarrow \begin{cases} n^r - r n & n \geq 0 \\ -n^r - r n & n \leq 0 \end{cases}$$



$$\rightarrow R_f = \mathbb{R}$$

