

$$f_{(-n)} - n(f_{(n)}) = r|n|+1 \xrightarrow{n \rightarrow \infty} n f_{(-n)} - n^r f_{(n)} = n(r|n|+1) \left\{ \begin{array}{l} -n^r f_{(n)} - f_{(n)} \\ = r n |n| + n - r |n| - 1 \end{array} \right.$$

$$f_{(n)} + n(f_{(-n)}) = r|-n|+1 = r|n|+1$$

$$\rightarrow -f_{(n)}(n^r+1) = r|n|(n-1) + (n-1) \rightarrow f_{(n)} = -\frac{(n-1)(r|n|+1)}{n^r+1}$$

$$f_{(n)} + f_{(0)} = n^r - rn + r \rightarrow r f_{(0)} = r \rightarrow f_{(0)} = 1$$

$$\rightarrow f_{(n)} = n^r - rn + r \rightarrow f_{(n^r)} = n^r - r n^r + r$$

$$y = \frac{n^r + n + r}{n-1} \rightarrow y n - y = n^r + n + r \rightarrow n^r + (1-y)n + (r+y) = 0$$

$$\rightarrow n = \frac{y+1 \pm \sqrt{(1-y)^2 - 4(r+y)}}{2} \rightarrow (1-y)^2 - 4(r+y) \geq 0 \rightarrow y^2 - 4y - 4 \geq 0$$

$$\rightarrow R_f = (-\infty, -1] \cup [4, +\infty)$$

$$الف) y = rn^r - \delta n + 1 \xrightarrow{a > 0} \begin{cases} \frac{b}{ra} = \frac{\delta}{\varepsilon} \rightarrow y_{\min} = \frac{-14}{n} \rightarrow R_f = \left[\frac{14}{n}, +\infty \right) \end{cases}$$

$$ب) y = \sqrt{-n^r + 4n - 5} \xrightarrow{a < 0} y_{\max} = 4 \rightarrow R_f = \sqrt{(-\infty, 4]} = [0, 4]$$

$$ج) y = \sqrt{n^r - 4n^r + 4n + 1} \rightarrow R_f = \sqrt{IR} = [0, +\infty)$$

$$د) y = \log(n^r - 4n^r + 4n + 1) \rightarrow R_{(n^r - 4n^r + 4n + 1)} = IR \xrightarrow{\log} R_f = (-\infty, +\infty)$$

$$هـ) \frac{r n + 1}{r n - 7} \xrightarrow{\text{تجزیه کسری}} R_f = IR - \left\{ \frac{a}{\varepsilon} \right\} = IR - \left\{ \frac{r}{7} \right\}$$

$$و) y = \sqrt{\frac{r n + 1}{n - r}} \rightarrow R_f = \sqrt{IR - r} = [0, +\infty) - \left\{ r \right\}$$

$$ز) y = \sqrt{n^r + r} + \frac{1}{\sqrt{n^r + r}} \rightarrow y_{\min} = \sqrt{r} + \frac{1}{\sqrt{r}} = \frac{\varepsilon}{\sqrt{r}} \rightarrow R_f = \left[\frac{\varepsilon}{\sqrt{r}}, +\infty \right)$$

$$ح) y = \left(n^r + \varepsilon + \frac{1}{n^r + \varepsilon} \right) - \varepsilon \rightarrow R_f = \left[\frac{1}{\varepsilon}, +\infty \right)$$

$$y = \frac{r n^r + \varepsilon}{n^r - 1} \rightarrow y n^r - y = r n^r + \varepsilon \rightarrow (y - r) n^r - (y + \varepsilon) = 0$$

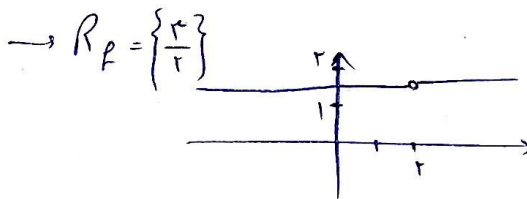
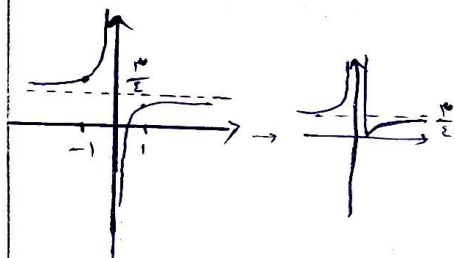
$$\rightarrow n = \frac{\pm \sqrt{r(y+\varepsilon)(y-r)}}{r(y-r)} \rightarrow \begin{cases} y \neq r \\ \varepsilon(y+\varepsilon)(y-r) \geq 0 \rightarrow \frac{-\varepsilon}{r} \leq y \leq \frac{-\varepsilon}{r} \end{cases}$$

$$\rightarrow R_f = \mathbb{R} - (-\varepsilon, r]$$

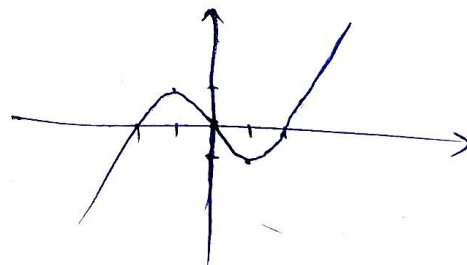
$$n^r = \square \rightarrow \begin{cases} n=0 \rightarrow y=-\varepsilon \\ n \rightarrow \infty \rightarrow y=r \end{cases} \xrightarrow{\text{مخرج/مقسوم علیه}} R_f = (-\infty, -\varepsilon] \cup (r, +\infty)$$

$$f(n) = \frac{r \sin n + \varepsilon}{\sin n - r} \rightarrow \begin{cases} \sin n = 1 \rightarrow y = -r \\ \sin n = -1 \rightarrow y = -\frac{1}{r} \end{cases} \xrightarrow{\text{مخرج/مقسوم علیه}} R_f = [-r, -\frac{1}{r}]$$

$$y = \left| \frac{r n - 1}{\varepsilon n} \right| \rightarrow R_f = [0, +\infty) \quad \left\{ \begin{aligned} y &= \sqrt{\frac{r n - 1}{\varepsilon n - 1}} = \sqrt{\frac{r}{\varepsilon}} \quad n \neq r \\ &\rightarrow R_f = \left\{ \frac{r}{r} \right\} \end{aligned} \right.$$



$$y = n|n| - r n \rightarrow \begin{cases} n^r - r n & n \geq 0 \\ -n^r - r n & n \leq 0 \end{cases}$$



$$\rightarrow R_f = \mathbb{R}$$

