

$$\begin{aligned} f(-x) - x f(x) &= 2|x| + 1 \xrightarrow{x \neq 0} x f(-x) - x^2 f(x) = 2x|x| + x \\ f(x) + x f(-x) &= 2|(-x)| + 1 \quad \text{چون این دو هم ندرست} \\ -x^2 f(x) - f(x) &= 2x|x| + x \Rightarrow f(x)(-x^2 - 1) = 2x|x| + x \rightarrow f(x) = \frac{2x|x| + x}{-x^2 - 1} \end{aligned}$$

صفحه آخر برگشتن

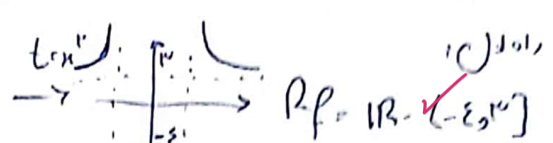
$$\begin{aligned} f(x) + f(0) &= x^2 - 3x + 4 \xrightarrow{x=0} 2f(0) = 4 \quad f(0) = 2 \quad f(x) = x^2 - 3x + 4 \\ &= x^2 - 3x + 2 \\ \rightarrow f(x^2) &= x^4 - 3x^2 + 2 \quad \checkmark \end{aligned}$$

$$\begin{aligned} y &= \frac{x^2 + x + 2}{x - 1} \rightarrow x^2 + x + 2 = yx - y \rightarrow x^2 + x(1 - y) + (2 + y) = 0 \\ x &= \frac{y - 1 \pm \sqrt{(y - 1)^2 - 4(y + 2)}}{2} \rightarrow y^2 - 4y + 1 - 4y - 8 \geq 0 \quad y^2 - 4y - 7 \geq 0 \\ &\quad (y - 7)(y + 1) \geq 0 \\ \begin{array}{c|c} -1 & 7 \\ \hline + & - & + \end{array} \quad R_f &= (-\infty, -1] \cup [7, +\infty) \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{a)} \quad y &= 2x^2 - 5x + 1 \quad x_s = \frac{-b}{2a} = \frac{-(-5)}{2(2)} = \frac{5}{4} \quad y_s = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 1 = -\frac{9}{4} \rightarrow \cup \\ R_f &= \left[-\frac{9}{4}, +\infty\right) \\ \Rightarrow y &= \sqrt{-x^2 + 4x - 5} \quad x_s = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = -2 \quad y_s = \sqrt{-9 + 16 - 5} = \sqrt{2} = 2 \rightarrow R_f = [2, 2] \\ \text{c)} \quad y &= \sqrt{x^2 - 3x + 5} + 1 \rightarrow \sqrt{12} = 2 \quad R_f = [0, +\infty) \\ \text{d)} \quad y &= \log(\underbrace{x^3 - 5x^2 + 4x + 6}_{\mathbb{R}^+ \rightarrow \mathbb{R}^+}) \quad \text{f} \rightarrow R_f = \mathbb{R} \quad \checkmark \end{aligned}$$

$$\begin{aligned} \text{a)} \quad y &= \frac{x^2 + 1}{x^2 - 2} \quad \mathbb{R} - \left\{\frac{2}{x}\right\} \rightarrow R_f = \mathbb{R} - \left\{\frac{2}{x}\right\} \\ \Rightarrow y &= \sqrt{\frac{x^2 + 1}{x^2 - 2}} \rightarrow R_f = [0, +\infty) - \left\{\frac{2}{x}\right\} \\ \text{c)} \quad y &= \frac{x^2 + 2}{\sqrt{x^2 + 2}} = \sqrt{x^2 + 2} + \frac{1}{\sqrt{x^2 + 2}} \quad \min_{x=0} \sqrt{2} + \frac{1}{\sqrt{2}} = \frac{3}{\sqrt{2}} \quad R_f = \left[\frac{3}{\sqrt{2}}, +\infty\right) \\ \text{d)} \quad y &= x^2 + \frac{1}{x^2 + 2} = x^2 + 2 + \frac{1}{x^2 + 2} - 2 \quad \min_{x=0} y = 2 + \frac{1}{2} - 2 = \frac{1}{2} \quad R_f = \left[\frac{1}{2}, +\infty\right) \end{aligned}$$

$$y = \frac{x^2 + \varepsilon}{x^2 - 1} \quad x^2 = t \quad y = \frac{t + \varepsilon}{t - 1}$$



$$R_f = \mathbb{R} - (-\varepsilon, \varepsilon]$$

(2)

$$\frac{x^2 + \varepsilon}{x^2 - 1} = y \rightarrow -\frac{y - \varepsilon}{y - 1} = x^2$$

$$x^2 = \frac{y + \varepsilon}{y - 1} \rightarrow \frac{-\varepsilon - 1}{+1 - 1} = \frac{-\varepsilon - 1}{0}$$

$$\begin{aligned} \min_{x^2} &= 0 \rightarrow y = -\varepsilon \\ \max_{x^2} &= +\infty \rightarrow y = 1 \end{aligned}$$

$$f(x^2) = \pm 1 \rightarrow \frac{y + \varepsilon}{y - 1} = \pm 1$$

$$R_f = \mathbb{R} - (-\varepsilon, 1]$$

if x=1

6

$$\sin x \xrightarrow{\min} -1 \rightarrow f(-1) = \frac{(-1) + \varepsilon}{-1 - 1} = \frac{\varepsilon}{-2} = -\frac{\varepsilon}{2}$$

$$\xrightarrow{\max} 1 \rightarrow f(1) = \frac{1 + \varepsilon}{1 - 1} = \frac{1}{-1} = -1 \rightarrow R_f = [-\frac{\varepsilon}{2}, -1]$$

(2)

7

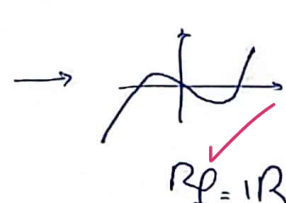
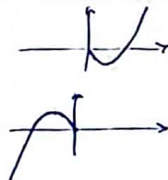
$$\text{الف) } y = \left| \frac{x-1}{\varepsilon} \right| \rightarrow R_f = [0, +\infty)$$

$$\Rightarrow y = \sqrt{\frac{x-1}{-1+\varepsilon}} = \sqrt{\frac{x-1}{1-\varepsilon}} = \frac{x}{1} \rightarrow R_f = \left[\frac{\varepsilon}{1}, \frac{1}{1} \right]$$

(2)

8

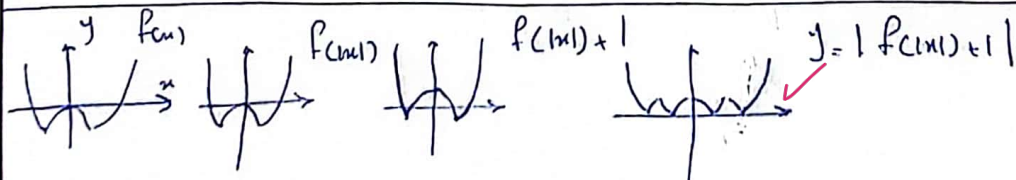
$$y = x|x| - k_n \xrightarrow{x \geq 0} x^2 - k_n \xrightarrow{x < 0} -x^2 - k_n$$



$$R_f = \mathbb{R}$$

(2)

9



$$y = |f(x)| + 1$$

(2)

10

۱- جای n ، $-n$ قرار می دهیم و یک دستگاه می سازیم:

$$\begin{aligned} f(-n) - n f(n) &= 2|n| + 1 \xrightarrow{\times n} \begin{cases} n f(-n) - n^2 f(n) = 2n|n| + n & \text{(I)} \\ f(n) + n f(-n) = 2|n| + 1 & \text{(II)} \end{cases} \\ f(n) + n f(-n) &= 2|n| + 1 \end{aligned}$$

$$\underline{\text{II} - \text{I}} \rightarrow (1 + n^2) f(n) = 2(1-n)|n| + 1 - n \Rightarrow f(n) = \frac{(1-n)(2|n|+1)}{1+n^2}$$