

$$\begin{aligned} f(-x) - x f(x) &= 2|x| + 1 \xrightarrow{x \neq 0} x f(-x) - x^2 f(x) = 2x|x| + x \\ f(x) + x f(-x) &= 2|(-x)| + 1 \\ -x^2 f(x) - f(x) &= 2|x| + 1 \Rightarrow f(x)(-x^2 - 1) = 2x|x| + 1 \rightarrow f(x) = \frac{2x|x| + 1}{-x^2 - 1} \end{aligned}$$

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$$\begin{aligned} f(x) + f(0) &= x^2 - 3x + 4 \xrightarrow{x=0} 2f(0) = 4 \quad f(0) = 2 \quad f(x) = x^2 - 3x + 4 - 2 \\ &= x^2 - 3x + 2 \\ \rightarrow f(x^2) &= x^4 - 3x^2 + 2 \end{aligned}$$

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$$\begin{aligned} y &= \frac{x^2 + x + 1}{x - 1} \rightarrow x^2 + x + 1 = yx - y \rightarrow x^2 + x(1 - y) + (1 + y) = 0 \\ x &= \frac{y - 1 \pm \sqrt{(y - 1)^2 - 4(y + 1)}}{2} \rightarrow y^2 - 4y + 1 - 4y - 4 \geq 0 \quad y^2 - 8y - 3 \geq 0 \\ &\quad (y - 5)(y + 1) \geq 0 \\ \begin{array}{c|c} -1 & 5 \\ \hline + & - & + \end{array} \quad R_f &= (-\infty, -1] \cup [5, +\infty) \end{aligned}$$

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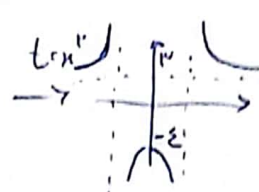
$$\begin{aligned} \text{a)} \quad y &= 2x^2 - 5x + 1 \quad x_s = \frac{-b}{2a} = \frac{-(-5)}{2(2)} = \frac{5}{4} \quad y_s = 2\left(\frac{5}{4}\right)^2 - 5\left(\frac{5}{4}\right) + 1 = -\frac{9}{4} \rightarrow \cup \\ R_f &= \left[-\frac{9}{4}, +\infty\right) \\ \text{b)} \quad y &= \sqrt{-x^2 + 4x - 5} \quad x_s = \frac{-b}{2a} = \frac{-(-4)}{2(-1)} = -2 \quad y_s = \sqrt{-9 + 10 - 5} = \sqrt{-4} = 2 \rightarrow R_f = [0, 2] \\ \text{c)} \quad y &= \sqrt{x^2 - 3x + 5} + 1 \rightarrow \sqrt{12} = 2 \quad R_f = [0, +\infty) \\ \text{d)} \quad y &= \log(\underbrace{x^3 - 5x^2 + 4x + 6}_{\mathbb{R}^+ \rightarrow \mathbb{R}^+}) \rightarrow R_f = \mathbb{R} \end{aligned}$$

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$$\begin{aligned} \text{a)} \quad y &= \frac{x+1}{x-2} \quad \mathbb{R} - \left\{\frac{2}{1}\right\} \rightarrow R_f = \mathbb{R} - \left\{\frac{3}{1}\right\} \\ \text{b)} \quad y &= \sqrt{\frac{x+1}{x-2}} \rightarrow R_f = [0, +\infty) - \{2\} \\ \text{c)} \quad y &= \frac{x^2 + 1}{\sqrt{x^2 + 1}} = \sqrt{x^2 + 1} + \frac{1}{\sqrt{x^2 + 1}} \quad \min_{x=0} \sqrt{1} + \frac{1}{\sqrt{1}} = 2 \quad R_f = \left[\frac{2}{\sqrt{1}}, +\infty\right) \\ \text{d)} \quad y &= x^2 + \frac{1}{x^2 + 1} = x^2 + \frac{1}{x^2 + 1} - \frac{1}{x^2 + 1} \quad \min_{x=0} y = \frac{1}{1} - \frac{1}{1} = 0 \quad R_f = \left[\frac{1}{1}, +\infty\right) \end{aligned}$$

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$$y = \frac{x^2 + \varepsilon}{x^2 - 1} \quad x^2 = t \quad y = \frac{t + \varepsilon}{t - 1}$$



$$R_f = \mathbb{R} - (-\varepsilon, \varepsilon]$$

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$$\frac{x^2 + \varepsilon}{x^2 - 1} = y \rightarrow -\frac{y - \varepsilon}{y - 1} = x^2 \quad x^2 = \frac{y + \varepsilon}{y - 1} \rightarrow \frac{-\varepsilon y}{y + \varepsilon - 1}$$

$$\begin{aligned} \min_{x^2} &= 0 \rightarrow y = -\varepsilon \\ \max_{x^2} &= +\infty \rightarrow y = 1 \end{aligned}$$

$$f(x) = \pm 1 \rightarrow \frac{y + \varepsilon}{y - 1} = \pm 1$$

$$R_f = \mathbb{R} - (-\varepsilon, 1]$$

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$$S_{\sin x} \xrightarrow{\min} -1 \rightarrow f(-1) = \frac{(-1) + \varepsilon}{-1 - 1} = \frac{\varepsilon}{-2} = -\frac{\varepsilon}{2}$$

$$\xrightarrow{\max} 1 \rightarrow f(1) = \frac{1 + \varepsilon}{1 - 1} = \frac{\varepsilon}{0} = +\infty \quad R_f = [-\frac{\varepsilon}{2}, +\infty)$$

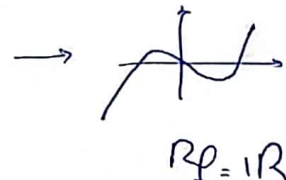
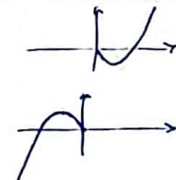
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$$\text{الف) } y = \left| \frac{x^2 - 1}{\varepsilon x} \right| \quad \text{Graphs showing the function and its range } R_f = (0, +\infty)$$

$$\Rightarrow y = \sqrt{\frac{x^2 - 1}{-1 + \varepsilon x}} = \sqrt{\frac{x(x - 1)}{x(1 - \varepsilon)}} = \frac{x}{1 - \varepsilon} \quad R_f = \left\{ \frac{x}{1 - \varepsilon} \right\}$$

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$$y = x|x| - kx \quad \xrightarrow{x \geq 0} x^2 - kx \quad \xrightarrow{x < 0} -x^2 - kx$$



$$R_f = \mathbb{R}$$

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$$y = |f(x)| + 1 \quad \text{Graphs showing the function and its range } R_f = [1, +\infty)$$

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