

(1)

$$f(-x) - xf(x) = 2|x| + 1$$

$$\rightarrow \text{مماثلہ} \rightarrow f(x) - (-x)f(-x) = 2|-x| + 1 \quad , \quad |-x| = x$$

$$\Rightarrow f(x) + xf(-x) = 2|x| + 1$$

$$A = f(-x), B = f(x) \Rightarrow A - xB = 2|x| + 1, B + xA = 2|x| + 1$$

$$\Rightarrow A = 2|x| + 1 + xB \xrightarrow{\text{جائے}} B + x(2|x| + 1 + xB) = 2|x| + 1$$

$$B + x(2|x| + 1) + x^2 B = 2|x| + 1$$

$$\Rightarrow (1 + x^2)B = (2|x| + 1)(1 - x)$$

$$\Rightarrow \boxed{f(x) = \frac{(2|x| + 1)(1 - x)}{1 + x^2}}$$

$$f(x) + f(0) = x^2 - x + 7 \quad (P)$$

$$\rightarrow f(0) \Rightarrow f(0) + f(0) = 0 + 0 - 7 \Rightarrow 2f(0) = -7 \Rightarrow f(0) = -\frac{7}{2}$$

$$f(x) + \frac{7}{2} = x^2 - x + 7 \rightarrow f(x) = x^2 - x + \frac{7}{2}$$

$$\Rightarrow \boxed{f(x^2) = x^4 - x^2 + \frac{7}{2}}$$

$$y = \frac{x^2 + x + 7}{x - 1} \rightarrow y(x - 1) = x^2 + x + 7 \quad (M)$$

$$xy - y = x^2 + x + 7 \quad , \quad \Delta \geq 0 \rightarrow x^2 + (1 - y)x + (7 + y) \geq 0$$

$$(1 - y)^2 - 4(7 + y) \geq 0 \rightarrow y^2 - 4y - 27 \geq 0 \rightarrow (y - 9)(y + 3) \geq 0$$

$$\frac{1}{x-1} = \frac{1}{x-1} + \frac{1}{x-1} \rightarrow \boxed{Rf = (-\infty, -1] \cup [9, +\infty)}$$



الف) $y = 2x^2 - 4x + 1$ و $x_A = \frac{b}{2a} = \frac{1}{2}$

$\rightarrow y_{min} = 2\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) + 1 = \frac{-14}{1} \Rightarrow R_f = \left[\frac{-14}{1}, +\infty\right)$

ب) $y = \sqrt{-x^2 + 4x - 2}$ و $y \geq 0$ (بسته به سوال)

$-x^2 + 4x - 2 = 0 \Rightarrow (x^2 - 4x) - 2 = -\left[\left(x - \frac{4}{2}\right)^2 - \frac{16}{4}\right] - 2$

$\Rightarrow -\left(x - \frac{4}{2}\right)^2 + \frac{16}{4} - \frac{2}{1} = -\left(x - \frac{4}{2}\right)^2 + \frac{41}{4} \rightarrow \max_{x=2} = \frac{41}{4}$

$\Rightarrow y_{max} = \sqrt{\frac{41}{4}} = \frac{\sqrt{41}}{2}$ و $y_{min} = 0 \Rightarrow R_f = \left[0, \frac{\sqrt{41}}{2}\right]$

ج) $y = \sqrt{x^2 - 4x^2 + 4x + 1}$ و if $x \rightarrow \infty \Rightarrow y \rightarrow \infty$
 $\Rightarrow R_f = [0, +\infty)$

د) $y = \deg(x^4 - 4x^2 + 5x + 2)$

if $x^4 - 4x^2 + 5x + 2 = 0$ $\rightarrow$ معادله درجه ۴

$\Rightarrow R_f = R = (-\infty, +\infty)$

ه) $y = \frac{x_{n+1}}{x_{n-2}} \rightarrow x_{n-2} \neq 0 \Rightarrow n \neq 2$

$y(x_{n-2}) = x_{n+1} \rightarrow x_{n-2}y - 4y = x_{n+1} \rightarrow n(2y - 4) = 4y + 1$

$\rightarrow n = \frac{4y+1}{2y-4}$ و $2y-4=0 \rightarrow y = \frac{2}{1} \Rightarrow R_f = R - \left\{\frac{2}{1}\right\}$

(ب)

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$$y = \sqrt{\frac{\varepsilon n+1}{n-r}} \quad , y \geq 0$$

$$y^2 = \frac{\varepsilon n+1}{n-r} \Rightarrow y^2(n-r) = \varepsilon n+1$$

$$\Rightarrow y^2 n - r y^2 = \varepsilon n+1 \rightarrow n(y^2 - \varepsilon) = r y^2 + 1$$

$$\rightarrow n = \frac{r y^2 + 1}{y^2 - \varepsilon} \rightarrow y^2 - \varepsilon = 0 \rightarrow y = \pm \sqrt{\varepsilon} \xrightarrow{y \geq 0} y = \sqrt{\varepsilon}$$

$$\Rightarrow y = \sqrt{\varepsilon} \rightarrow R_f = [0, +\infty) - \{\sqrt{\varepsilon}\} = [0, \sqrt{\varepsilon}) \cup (\sqrt{\varepsilon}, +\infty)$$

$$ج) y = \frac{n^r + \varepsilon}{\sqrt{n^r + r}} \quad , n^r + \varepsilon = (n^r + r) + 1$$

$$\rightarrow y = \frac{n^r + r}{\sqrt{n^r + r}} + \frac{1}{\sqrt{n^r + r}} = \sqrt{n^r + r} + \frac{1}{\sqrt{n^r + r}}$$

$$\Rightarrow t = \sqrt{n^r + r} \quad , t^r \geq 0 \Rightarrow t \geq \sqrt{r}$$

$$\Rightarrow y = t + \frac{1}{t} \Rightarrow y_{\min} \xrightarrow{t=\sqrt{r}} t + \frac{1}{t} = \sqrt{r} + \frac{1}{\sqrt{r}}$$

$$\rightarrow y_{\min} = \sqrt{r} + \frac{1}{\sqrt{r}} = \frac{\varepsilon}{\sqrt{r}} \Rightarrow R_f = \left[\frac{\varepsilon}{\sqrt{r}}, +\infty \right)$$

$$٢) y = n^r + \frac{1}{n^r + \varepsilon} \quad , n^r + \varepsilon \geq 0 \Rightarrow t \geq 0$$

$$y = t + \frac{1}{t + \varepsilon} \quad , t \geq 0 \rightarrow 1 - \frac{1}{(t + \varepsilon)^r} \geq 0$$

$$\Rightarrow y_{\min} \xrightarrow{t=0} y_{\min} = 0 + \frac{1}{\varepsilon} = \frac{1}{\varepsilon}$$

$$\Rightarrow R_f = \left[\frac{1}{\varepsilon}, +\infty \right)$$

$$y = \frac{3n^2 + 4}{n^2 - 1} \rightarrow y n^2 - y = 3n^2 + 4$$

(۴)

$$\rightarrow (y-3)n^2 - y = 4 \rightarrow n^2 = \frac{y+4}{y-3}$$

$$\Rightarrow \frac{y+4}{y-3} \geq 0 \rightarrow y \leq -4, y \geq +3$$

$$\text{if } y \leq -4 \rightarrow \oplus / \text{if } y \geq -4 \rightarrow \ominus$$

$$\text{if } y \leq +3 \rightarrow \ominus / \text{if } y \geq +3 \rightarrow \oplus$$

$$\Rightarrow y \leq -4 \vee y \geq +3 \Rightarrow \text{domain}$$

$$R_f = (-\infty, -4] \cup [3, +\infty)$$

$$f(x) = \frac{x \sin x + 1}{\sin x - 1} \quad -1 \leq \sin x \leq +1$$

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$$\rightarrow \text{if } \sin x = t \Rightarrow -1 \leq t \leq +1 \Rightarrow y = \frac{t+1}{t-1}$$

$$\rightarrow \text{if } t = -1 \Rightarrow \frac{-4}{-2} = 2$$

$$\rightarrow \text{if } t = +1 \Rightarrow \frac{2}{0} = \infty$$

$$\Rightarrow R_f = [2, +\infty)$$

$$\text{ع۱) } y = \left| \frac{n-1}{n} \right| \Rightarrow \frac{n-1}{n} = \frac{n}{n} - \frac{1}{n}$$

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$$\rightarrow n \neq 0 \rightarrow \frac{n}{n} - \frac{1}{n} \rightarrow \frac{n-1}{n} \Rightarrow y \geq 0 \rightarrow R_f = [0, +\infty)$$

(ارائه)

$$\Rightarrow y = \sqrt{\frac{9n-18}{-2+2n}} \rightarrow y = \sqrt{\frac{9(n-2)}{2(n-1)}}$$

$$y \geq \frac{3}{2} \sqrt{\frac{n-2}{n-1}} \xrightarrow{\text{انتقال}} y \geq 0 \rightarrow t = \frac{n-2}{n-1}$$

$$t = 1 - \frac{1}{n-1} \xrightarrow{\text{مقادیر}} t \geq 0 \rightarrow t \in [0, 1) \cup (1, +\infty)$$

$$y \geq \frac{3}{2} \sqrt{t} \rightarrow 0 \leq y < \frac{3}{2} \vee y > \frac{3}{2}$$

$$\Rightarrow R_f = [0, \frac{3}{2}) \cup (\frac{3}{2}, +\infty)$$

$$y = x|x| - 2x \rightarrow \text{if } x \geq 0 \rightarrow y = x^2 - 2x$$

④

$$\Rightarrow y = x^2 - 2x = x^2 - 2x + 1 - 1 = (x-1)^2 - 1 \rightarrow y_{\min} \xrightarrow{x=1} -1$$

$$\Rightarrow y_{\min} = -1 \Rightarrow R_{f④} = [-1, +\infty)$$

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(ارائه)

$$\text{if } x < 0 \rightarrow |x| = -x \Rightarrow y = -x^2 - 2x = -(x^2 + 2x)$$

$$\Rightarrow y = -(x^2 + 2x) = -[(x+1)^2 - 1] = 1 - (x+1)^2$$

$$\rightarrow y_{\max} = 1 \rightarrow R_{f⑤} = (-\infty, 1]$$

$$\rightarrow R_{f④} \cup R_{f⑤} = (-\infty, 1] \cup [-1, +\infty) = \mathbb{R} = R_f$$

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شنبه

۶ جمادی الاول ۱۴۴۸

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$$y = f(|x| + 1)$$

$$f(x) \xrightarrow{1/x} f(|x| + 1), f(|x| + 1)$$

(۱۰)

(?)