

$$f(-x) - xf(x) - 2|x| + 1 \quad x \rightarrow -x \rightarrow f(x) + xf(-x) - 2|x| + 1$$

$$2|x| + 1 - A \rightarrow f(-x) - xf(x) - A \rightarrow f(-x) - A - xf(x)$$

$$f(x) + xf(-x) - A$$

$$\rightarrow f(x) + Ax + x^2 f(x) - 1 \quad \left(\rightarrow f(x) - \frac{(1-x)(2|x|+1)}{x^2+1} \right)$$

$$f(x) + f(0) - x^2 - 3x + 6 \quad x=0 \rightarrow 2f(0) - 6 \rightarrow f(0) = 3$$

$$f(x) - x^2 - 3x + 3 \rightarrow x \cdot x \left(f(x^2) - x^2 - 3x^2 + 3 \right)$$

$$y \cdot \frac{x^2 + x + 1}{x-1} \rightarrow yx - y - x^2 + x + 1 \rightarrow -x^2 + (y-1)x - y + 1 = 0$$

$$\Delta \geq 0 \rightarrow y^2 + 1 - 4y - 4y - 1 \rightarrow y^2 - 8y - 1 \geq 0$$

$$\rightarrow \frac{1}{+} \frac{1}{-} \rightarrow \mathbb{R}_y \cdot (-\infty, -1] \cup [7, +\infty)$$

الف) $y \cdot 2x^2 - 5x + 1 = 0 \rightarrow \left(\mathbb{R}_y \cdot \left[-\frac{1}{2}, +\infty \right) \right)$ ب) $y \cdot \sqrt{x^2 + 4x - 5}$

$$\rightarrow \frac{-b}{2a} = \frac{5}{4} \rightarrow y = \frac{1}{2} \quad \left(\mathbb{R}_y \cdot \left[-\frac{1}{2}, +\infty \right) \right) \rightarrow \frac{-b}{2a} = \frac{5}{4} \rightarrow y = \frac{1}{2}$$

ج) $y \cdot \sqrt{x^2 - 3x^2 + 4x + 1}$ د) $y \cdot (x^3 - 6x^2 + 7x + 4)$

$$\rightarrow \sqrt{\text{تابع درجه ۳}} \rightarrow \mathbb{R} \rightarrow \left(\mathbb{R}_y \cdot [0, +\infty) \right) \rightarrow \log \mathbb{R}^+ \rightarrow \left(\mathbb{R}_y \cdot \mathbb{R} \right)$$

الف) $y \cdot \frac{3x+1}{2x-6}$ ب) $y \cdot \sqrt{\frac{4x+1}{x-2}}$

$$\rightarrow \mathbb{R}_y \cdot \mathbb{R} - \left\{ \frac{3}{2} \right\} \quad \mathbb{R}_y \cdot (0, +\infty] - \left\{ \frac{1}{2} \right\}$$

ج) $y \cdot \frac{x^2 + 4}{\sqrt{x^2 + 3}}$ د) $y \cdot \frac{x^2 + 1}{x^2 + 4} + \frac{4 - x^2}{x^2 + 4}$

$$\rightarrow \left[\frac{1}{2}, +\infty \right) \rightarrow \left(\mathbb{R}_y \cdot [6, +\infty) \right) \rightarrow \left[\frac{1}{2}, +\infty \right) + \left[\frac{1}{2}, +\infty \right) - 4$$

$$\left(\mathbb{R}_y \cdot [-2, +\infty) \right)$$

$$f(x) = \frac{x^2}{x^2+1} \rightarrow y = \frac{x^2}{x^2+1} \rightarrow y(x^2+1) = x^2 \rightarrow yx^2 + y = x^2 \rightarrow yx^2 - x^2 = -y \rightarrow x^2(y-1) = -y \rightarrow x^2 = \frac{-y}{y-1} = \frac{y}{1-y}$$

$$x = \sqrt{\frac{y}{1-y}} \rightarrow \text{Domain: } y \in [0, 1) \rightarrow x \in (-\infty, \infty)$$

$$f(x) = \frac{y \sin x + 1}{\sin x - 1} \rightarrow R \sin x = [-1, 1]$$

$$R_f = [-3, -\frac{1}{2}] \rightarrow \frac{1}{2} \rightarrow \sin x - 1 \neq 0$$

$$f(x) = \frac{x-1}{x^2} \rightarrow R = \left\{ \frac{1}{x} \right\} \rightarrow R_y = [1, \infty)$$

$$f(x) = \frac{x-1}{-1+x} \rightarrow \sqrt{12 - \left\{ \frac{1}{x} \right\}} \rightarrow [0, \infty) - \left\{ \frac{1}{x} \right\}$$

$$f(x) = x|x| - 2x$$

$$x \geq 0 \rightarrow x^2 - 2x \rightarrow (1, \infty) \rightarrow R = [-1, \infty)$$

$$x \leq 0 \rightarrow -x^2 - 2x \rightarrow (-1, 1) \rightarrow R = (-\infty, 1]$$

