

(1)

$$\begin{cases} f(n) - n f(n) = r(n+1) \xrightarrow{xn} n f(n) - n^r f(n) = n(r(n+1)) \\ f(n) + n(f(n)) = r(-n) + 1 = r(n+1) \end{cases}$$

$$\rightarrow -f(n)(n^r+1) = r(n)(n-1) + (n-1) \rightarrow f(n) = -\frac{(n-1)(r(n+1))}{n^r+1}$$

(2)

$$n=0 \rightarrow f(0) + f(0) = r \rightarrow \underline{f(0) = \frac{r}{2}}$$

$$\rightarrow f(n) = n^r - r n + r \rightarrow f(n^r) = n^{2r} - r n^r + r$$

$$y = \frac{n^r + n + r}{n-1} \rightarrow y n - y = n^r + n + r \rightarrow n^r + n(1-y) + r + y = 0$$

(3)

$$\rightarrow n = \frac{1+y \pm \sqrt{y^2 - 4y - r}}{r} \rightarrow y^2 - 4y - r \geq 0 \rightarrow R_f = (-\infty, -1] \cup [r, +\infty)$$

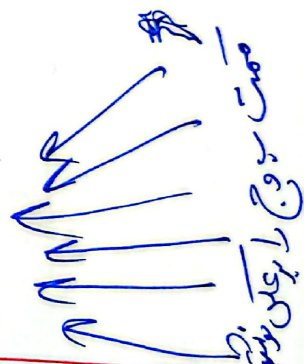
الف) $\text{ext} \left| \frac{n}{y} \right| \quad y_{\min} = -\frac{1}{r} \rightarrow R_f = [-\frac{1}{r}, +\infty)$

(4)

ب) $\text{ext} \left| \frac{n}{y} \right| \quad y_{\min} = r \rightarrow \sqrt{R_f} = \sqrt{[-\infty, r]} \rightarrow R_f = [0, r]$

ج) $y_{\min} = \sqrt{r} + \frac{1}{\sqrt{r}} = \frac{r}{\sqrt{r}} \rightarrow R_f = [\frac{r}{\sqrt{r}}, +\infty)$

د) $y = (n^r + r + \frac{1}{n^r+r}) - r \quad R_f = [\frac{1}{r}, +\infty)$



الف) $R_f = R - \left\{ \frac{r}{r} \right\}$

(5)

ب) $R_f = \sqrt{R - \{r\}} \rightarrow R_f = [0, +\infty) - \{r\}$

ج) $\sqrt{n^r - r n^r + n + 1} \rightarrow R_f = \sqrt{R} \rightarrow R_f = [0, +\infty)$



د) $y = \log(n^r - r n^r + n + r) \rightarrow \underline{R_f = R}$



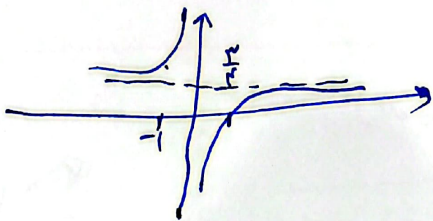
$$y = \frac{ru + r}{u^2 - 1} \rightarrow yu^2 - y = ru + r \rightarrow (y - r)u^2 = (y + r) \rightarrow \text{نقطه بحرانی} \quad (9)$$

$$u = \frac{\pm \sqrt{f(y+r)(y-r)}}{y(y-r)} \rightarrow \begin{cases} y \neq r \\ f(y+r)(y-r) \geq 0 \rightarrow \frac{-r}{+r} \leq \frac{r}{-r} \end{cases}$$

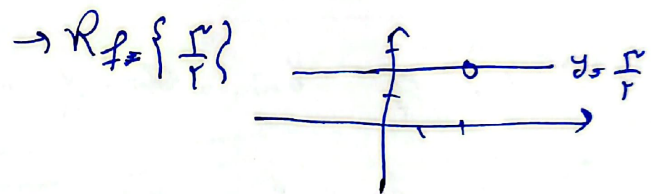
$$R_f = \mathbb{R} - (-r, r] \quad \left\{ \begin{array}{l} u=0 \rightarrow y=-r \\ u \rightarrow \infty \rightarrow y=r \end{array} \right. \quad \text{نقطه بحرانی} \quad R_f = \mathbb{R} - (-r, r]$$

$$f(u) = \frac{r \sin u + r}{\sin u - r} \rightarrow \begin{cases} \sin u = 1 \rightarrow y = -r \\ \sin u = -1 \rightarrow y = -\frac{1}{r} \end{cases} \rightarrow R_f = [-r, -\frac{1}{r}] \quad \text{نقطه بحرانی}$$

$$y = \left| \frac{ru - 1}{ru} \right| \rightarrow R_f = [0, \infty)$$

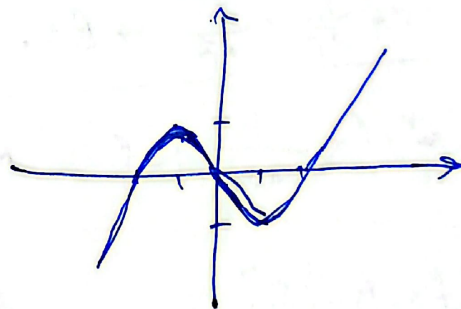


$$y = \sqrt{\frac{ru - r}{ru - 1}} = \sqrt{\frac{r}{r}} \quad u \neq r \quad (1)$$



$$y = u|u| - ru \begin{cases} ru - ru & u \geq 0 \\ -ru - ru & u \leq 0 \end{cases}$$

$$\rightarrow R_f = \mathbb{R}$$



$$y = f(u)$$

$$y = f(u)^{+1}$$

$$y = |f(u)^{+1}|$$

(10)

