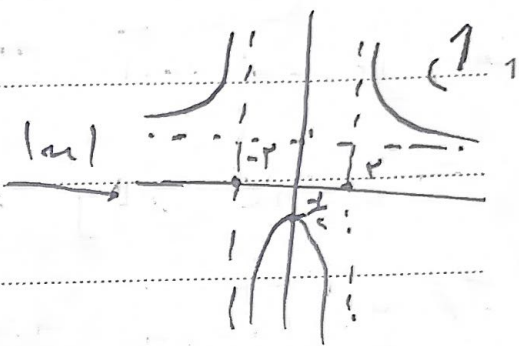
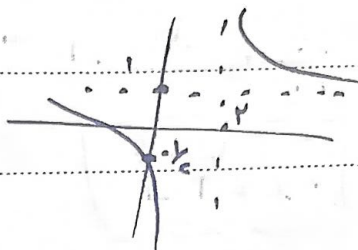


الف) $y = \frac{|x|+1}{|x|-2}$



$\rightarrow R_f: (-\infty, -\frac{1}{2}] \cup (1, +\infty)$

ب) $y = \frac{x^2+1}{|x|} = \frac{|x|+1}{|x|} = 1 + \frac{1}{|x|}$

$|x| > 0 \rightarrow R_f: [2, +\infty)$

$y = \frac{x^2+2x-5}{x+2} \rightarrow y(x+2) = x^2+2x-5 \rightarrow x^2+(2+y)x-5-2y = 0$ (10)

$\Delta = \frac{-(2+y)^2 - 4(-5-2y)}{4} \geq 0$

$\Rightarrow y^2+4y+12 \geq 0 \rightarrow (y+2)(y+6) \geq 0$

$\rightarrow R_f: (-\infty, -6] \cup [-2, +\infty)$

$R_f: x^2 - x + 1 \neq 0 \rightarrow \Delta = -3 \rightarrow \Delta < 0 \rightarrow R_f: \mathbb{R}$ (105)

تمرین: $\frac{(x-k)^r}{(x-\frac{1}{k})^r + \frac{r}{k}}$ $x-k=t \rightarrow \frac{t^r}{t^r + \frac{r}{k}}$ $y \Rightarrow t^r = \frac{-\frac{r}{k}y}{y-1} = \frac{-ry}{ry-k}$

$\rightarrow t^r \geq 0 \rightarrow \frac{-ry}{ry-k} \geq 0 \rightarrow \frac{1}{-\frac{1}{k} + \frac{1}{y}} \Rightarrow R_f: [0, 1)$

الف) $y = \sqrt{-x^2 + 6x - 1}$ $\xrightarrow{\text{max}} \frac{b}{2a} = \frac{-6}{-2} = 3 \xrightarrow{m=3} -x^2 + 6x - 1 = 0$ (1)

$\Rightarrow y = \sqrt{(-\infty, 6]} = [0, \sqrt{8}]$

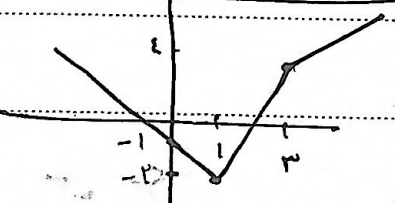
ب) $y = \sqrt{2x^2 - 4x}$ $\xrightarrow{\text{min}} \frac{b}{2a} = \frac{4}{4} = 1 \xrightarrow{m=1} 2x^2 - 4x = 0 \Rightarrow x = 0 \text{ or } 2$

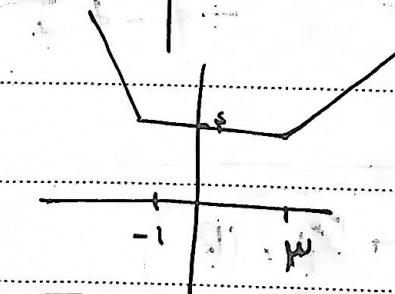
$\Rightarrow y = \sqrt{(-\infty, +\infty)} = [-2, +\infty)$ 5

الذ) $x^3 - x + 1$ $\rightarrow R: (-\infty, +\infty)$ (4)

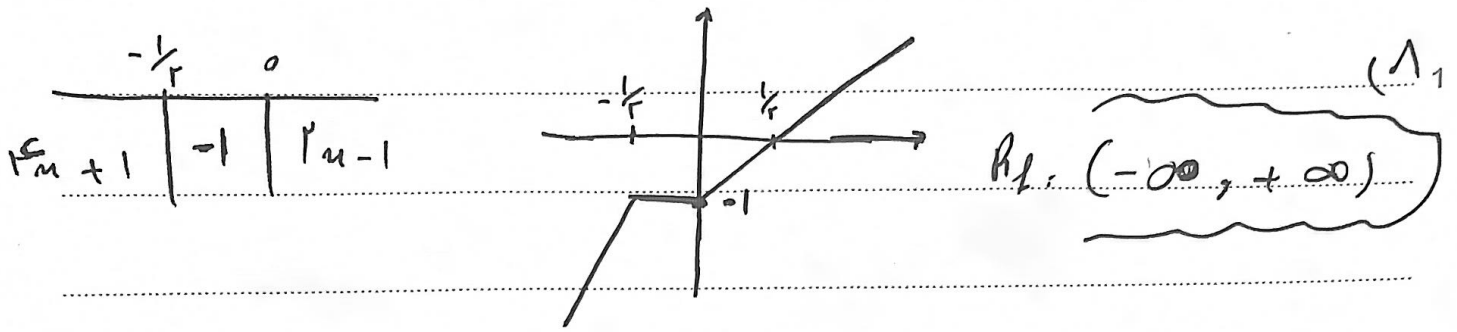
$\rightarrow f_1(x) = (x^3 - x + 1)$ $\rightarrow f_2(x) = f_1(0, +\infty) = (-\infty, +\infty)$

ب) $\frac{x^2+1}{x} = x + \frac{1}{x}$ $\xrightarrow{x>0} x + \frac{1}{x} \geq 2 \Rightarrow f_1[2, +\infty) = [1, +\infty)$ 10

الف)  $\Rightarrow R_f: [-2, +\infty)$ (6)

ب)  $\rightarrow R_f: [1, +\infty)$ 15

$-1 \leq x \leq 2$ $\Rightarrow R_f: (-\infty, -\frac{1}{2}] \cup [-\frac{1}{2}, +\infty)$ (7)



الف) $y = [m] + [-n] + r$ (95)

$m \in \mathbb{Z} \rightarrow y = r$
 $m \notin \mathbb{Z} \rightarrow y = r$

$\Rightarrow R_f: \{r\}$

ب) $-2 < m < -1 \rightarrow m+2$

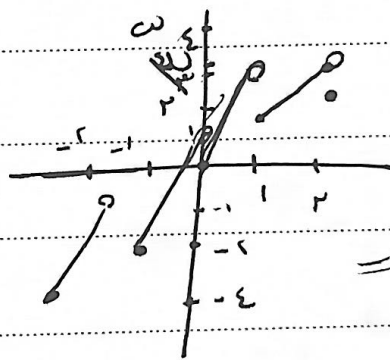
$-1 < m < 0 \rightarrow m+1$

$0 < m < 1 \rightarrow m$

$1 < m < 2 \rightarrow m-1$

~~$m=2 \rightarrow m-2$~~

$m=2 \rightarrow (m+2)-2 = 2$



$\Rightarrow R_f = [-2, \infty)$

10

الف) $\begin{matrix} \text{sinus} \rightarrow 1-1+1=1 \\ \text{cosinus} \rightarrow 1+1+1=3 \\ \text{tanus} \rightarrow \frac{1}{2}-\frac{1}{2}+1=\frac{3}{2} \end{matrix}$ $\Rightarrow R_f: [\frac{3}{2}, 3]$ (10)

15

ب) $\sin^2 z \rightarrow z^2 + z + 1$

$\begin{matrix} z=1 \rightarrow 1+1+1=3 \\ z=0 \rightarrow 0+0+1=1 \\ z=-\frac{1}{2} \rightarrow \frac{1}{4}-\frac{1}{2}+1=\frac{3}{4} \end{matrix}$

$\Rightarrow R_f: [1, 3]$