

الف) $y = \frac{|x|+1}{|x|-2}$
 $\frac{aD+b}{cD+d}$ ✓
 $\max_{|x| \rightarrow +\infty} y \rightarrow 1 \Rightarrow R_f = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$
 $\min_{|x|=0} y = -\frac{1}{2}$
 $|x|-2=0 \rightarrow$ قابلیت ضرب
 $\sqrt{\text{شون منفی را ندارد}}$

ب) $y = \frac{x^2+1}{|x|}$
 $y = |x| + \frac{1}{|x|}$
 $|x| + \frac{1}{|x|} \geq 2 \Rightarrow R_f = [2, +\infty)$

$y = \frac{x^2+2x-2}{x+3}$
 $\downarrow$
 $x^2 + (2-y)x - 2 - 3y = 0$
 $x = \frac{y-2 \pm \sqrt{(y-2)^2 - 4(-2-3y)}}{2} \rightarrow y^2 + 4y + 12 \geq 0$
 $(y+2)(y+6) \geq 0$
 $\frac{-4}{1} \leq y \leq \frac{-6}{1} \rightarrow R_f = (-\infty, -4] \cup [-2, +\infty)$

$f(x) = \frac{x^2-x+\frac{1}{2}}{x^2-x+1} = 1 - \frac{\frac{1}{2}}{x^2-x+1}$
 $\downarrow$
 $x^2-x+1 \neq 0$
 $\Delta = -3 < 0$ ✓
 $x^2-x+1 \xrightarrow{x \rightarrow \pm\infty} [\frac{1}{2}, +\infty) \rightarrow -\frac{1}{2(x^2-x+1)} \xrightarrow{x \rightarrow \pm\infty} [-1, 0)$
 $1 - \frac{1}{2(x^2-x+1)} \rightarrow R_f = [0, 1)$
 $D_f = \mathbb{R}$

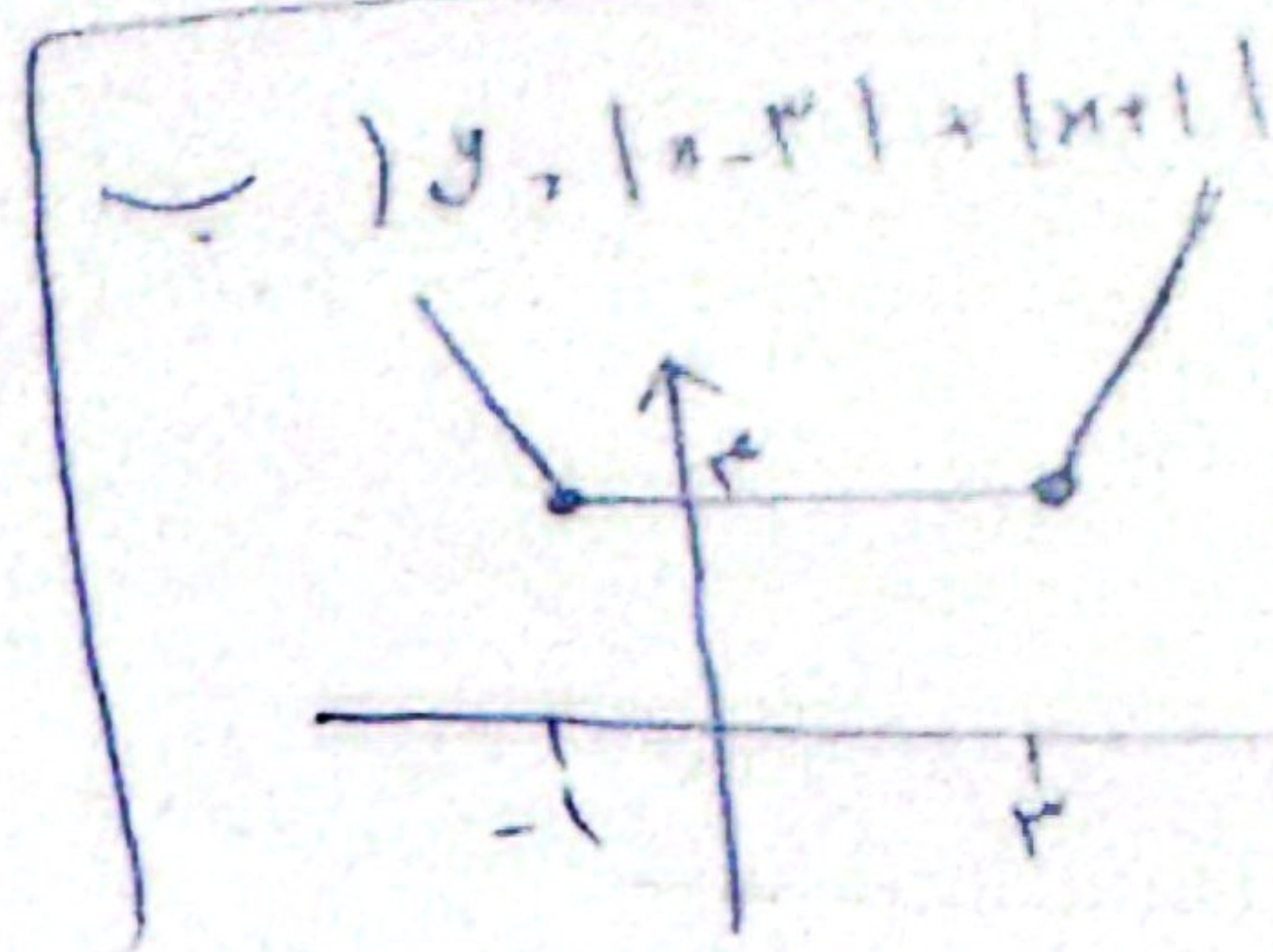
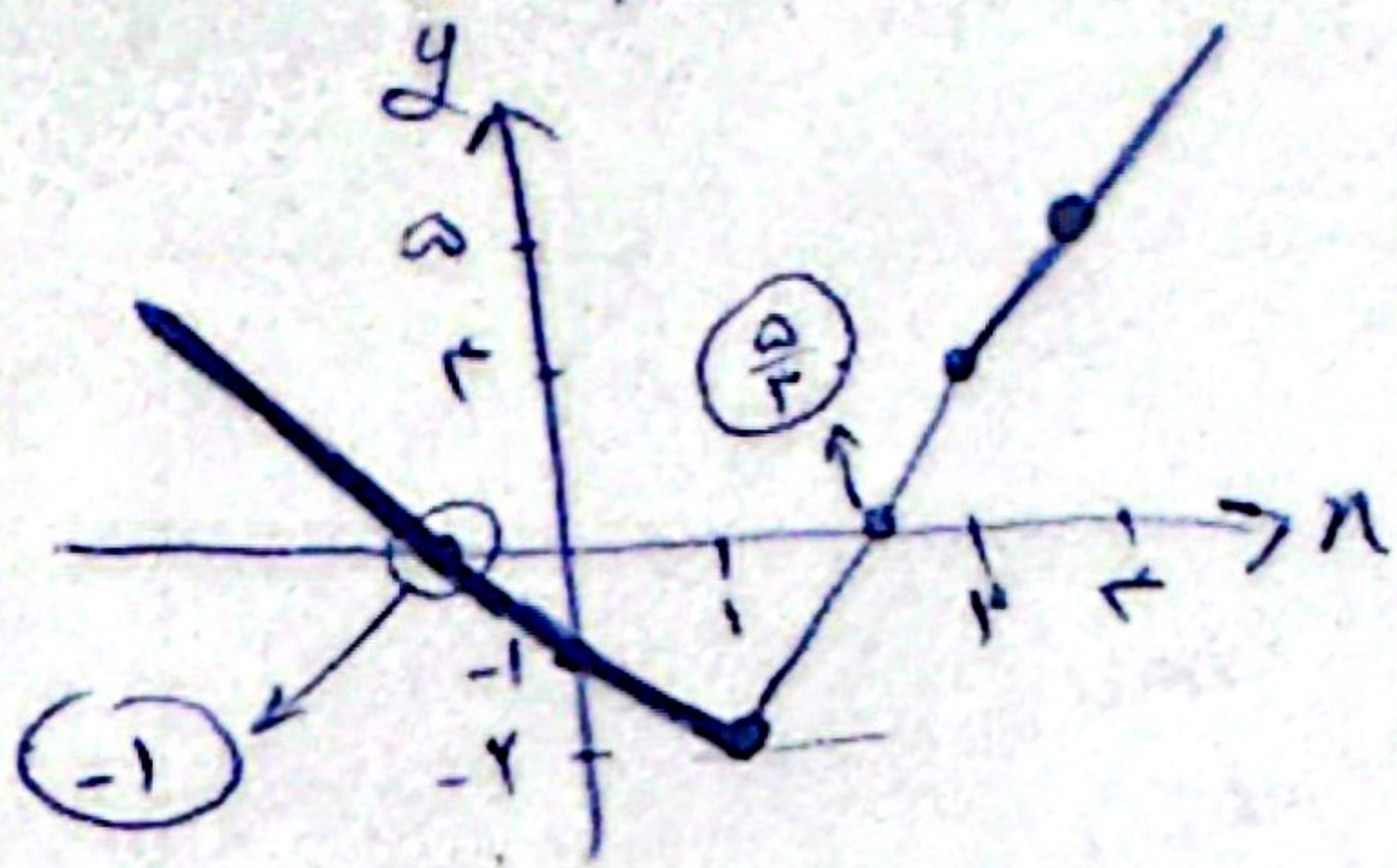
الف) $y = \sqrt{-x^2+4x-1} = \sqrt{(-\infty, 3]} \rightarrow R_f = [0, \sqrt{3}]$
 $-x^2+4x-1 \xrightarrow{x \rightarrow \pm\infty} (-\infty, 3]$

ب) $y = \sqrt[3]{x^2-4x} = \sqrt[3]{[-\infty, +\infty)} \rightarrow R_f = [-2, +\infty)$
 $x^2-4x \xrightarrow{x \rightarrow \pm\infty} [-\infty, +\infty)$

الف) $y = \frac{x^2-x+1}{x^2-x+1} \rightarrow y = 1 \rightarrow R_f = \mathbb{R}$
 $\frac{x^2-x+1}{x^2-x+1} \xrightarrow{x \rightarrow \pm\infty} 1$

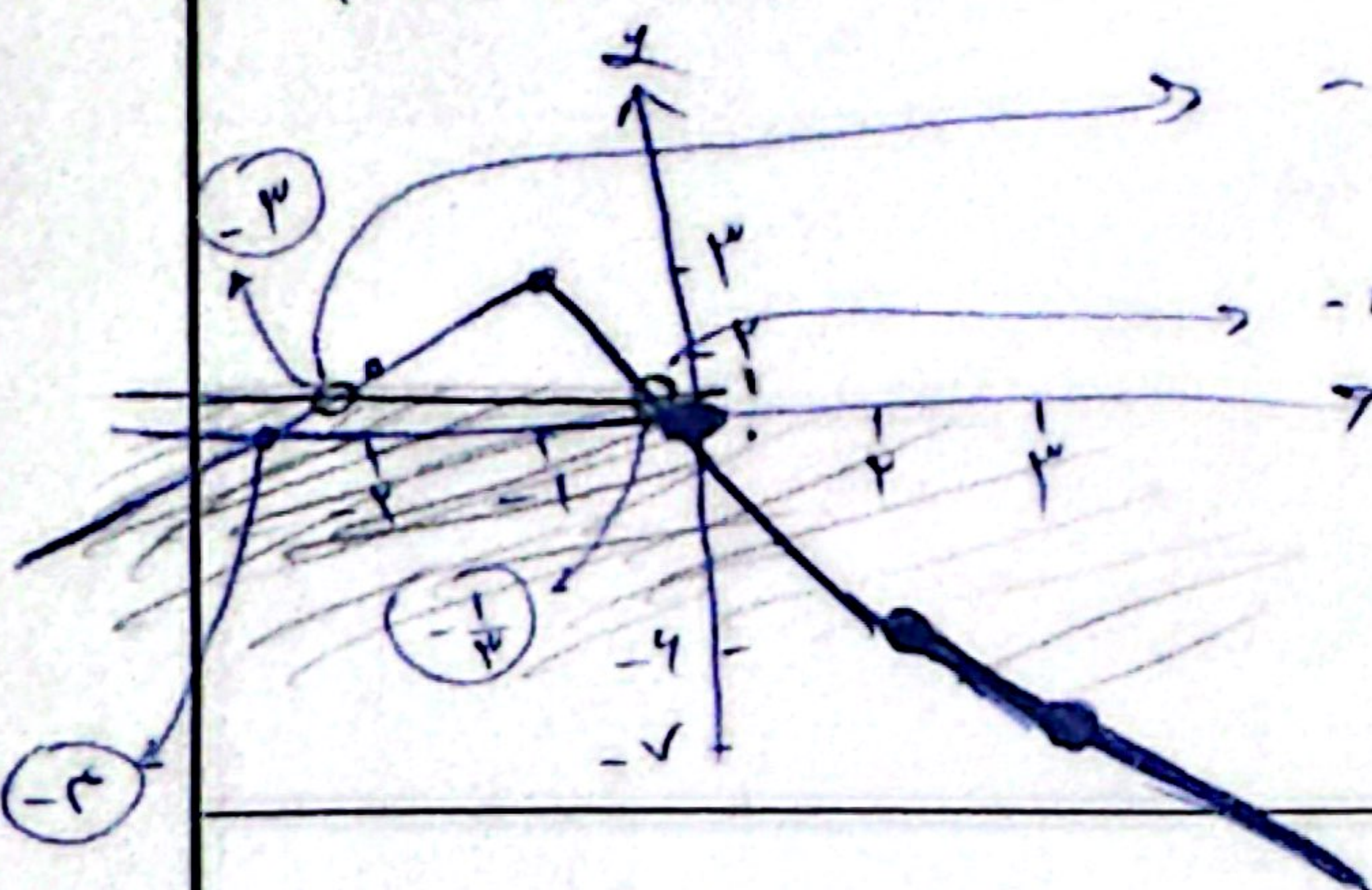
ب) $y = \frac{x^2+1}{x} \rightarrow y = \frac{x}{1} \rightarrow R_f = [1, +\infty)$
 $\frac{x^2+1}{x} \xrightarrow{x \rightarrow \pm\infty} \frac{x}{1}$

ii) $y = |x-2| - |x-3| \rightarrow R_f = [1, +\infty)$



$R_f = [1, +\infty)$

$|x-2| - |x+1| \leq 1$



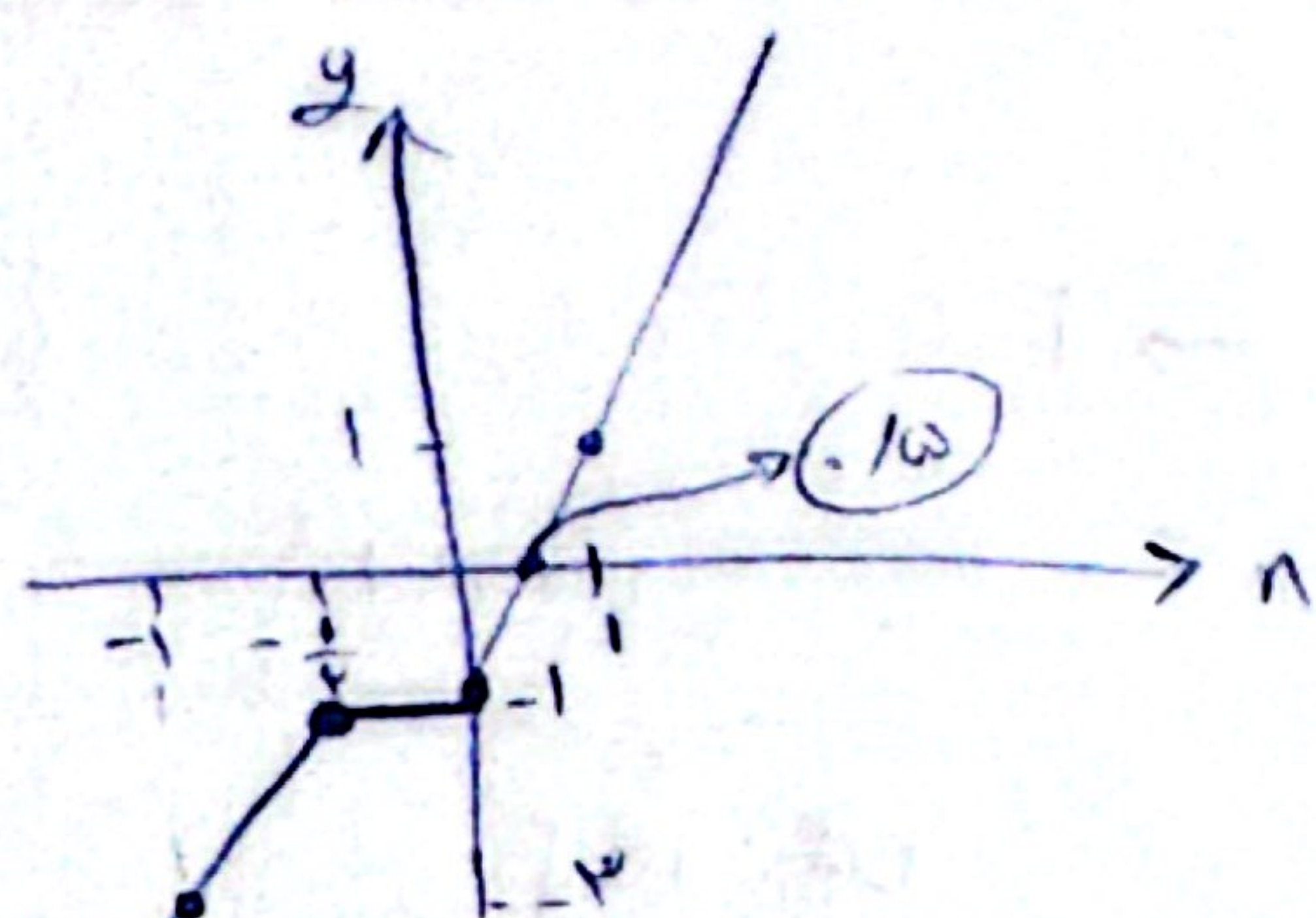
$-x+2+x+1 \leq 1$
 $x \leq -2$

$-x+2-x-1 \leq 1$
 $x \geq -1$

$\rightarrow x \in (-\infty, -2] \cup [-1, +\infty)$

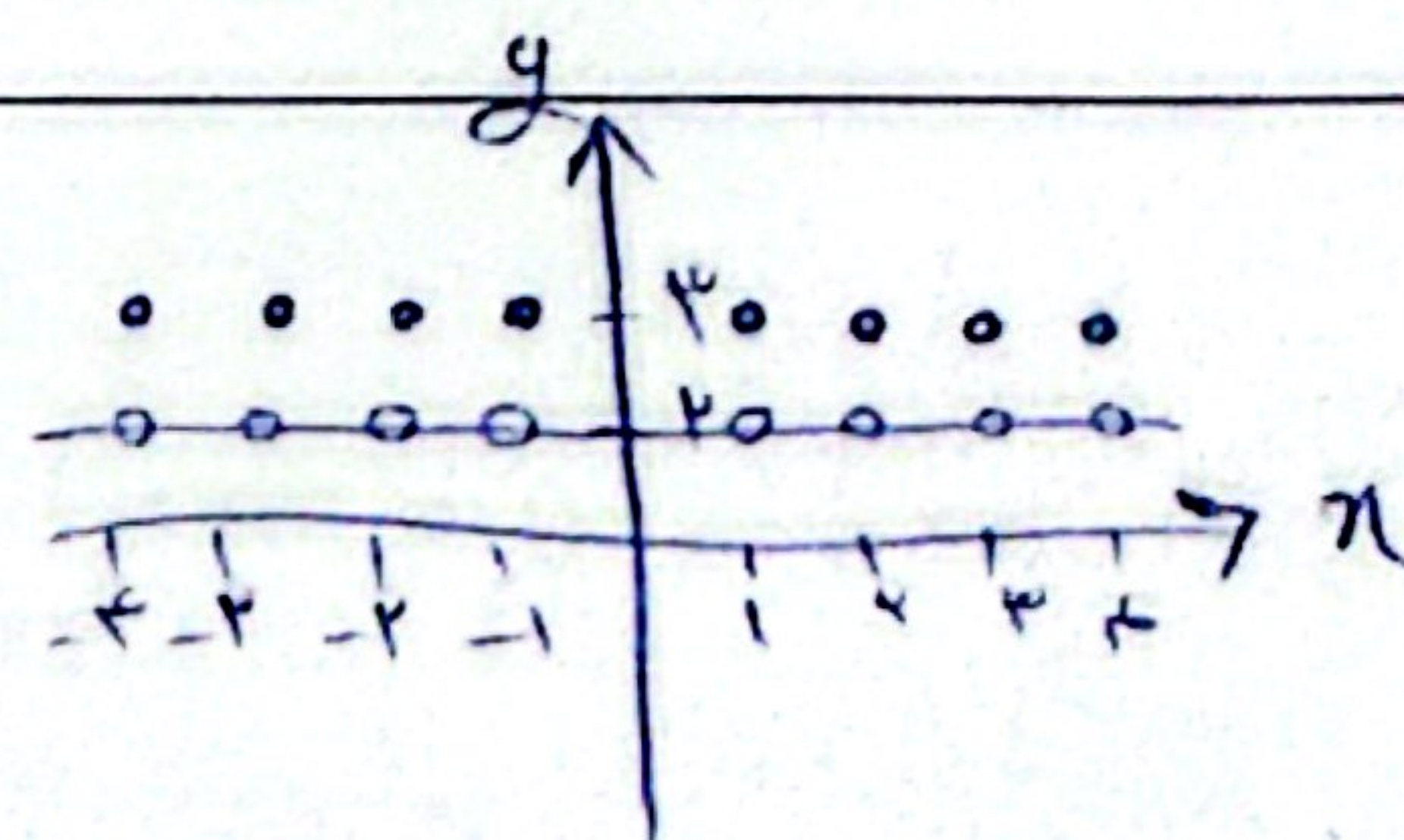
$y = x - |x+1| + |x|$

$\rightarrow R_f = \mathbb{R}$



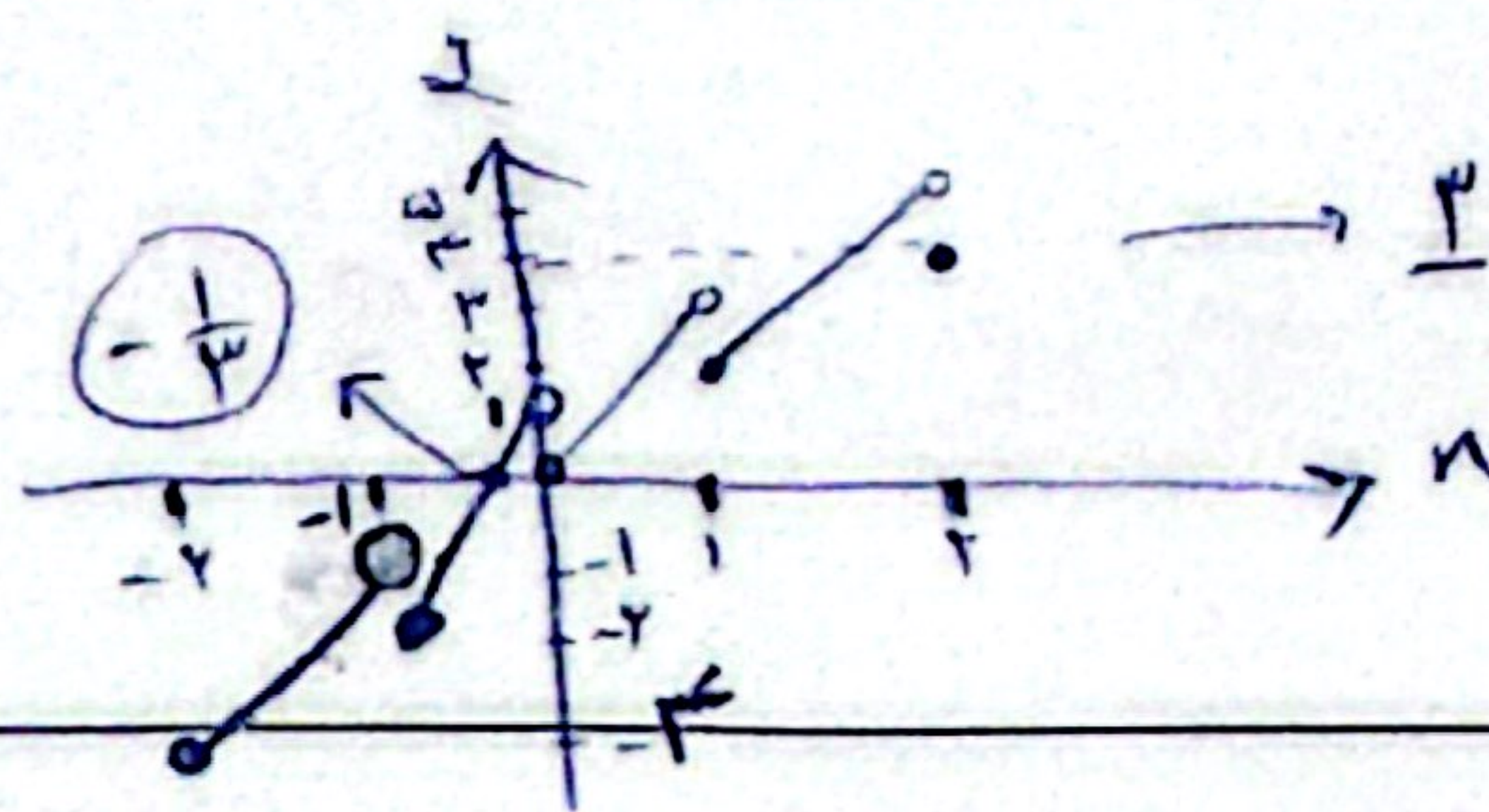
iii) $y = [x] + [3-x] = [x] + [-x] + 3$

$R_f = \{1, 2, 3\}$



iv) $y = x - [x]$

$-1 \leq x < -1$ $x+2$
 $-1 \leq x < 0$ $x+1$
 $0 \leq x < 1$ x
 $1 \leq x < 2$ $x-1$
 $x \geq 2$ $x-2$



$f: \mathbb{R} \rightarrow [0, 1)$

$\rightarrow R_f = [0, 1)$

v) $y = \sin^2 x - \sin x + 1$

$\sin x = 1, -1, \frac{1}{2} \Rightarrow R_f = [\frac{3}{4}, 3]$

$y = 3, 3, \frac{3}{4}$

vi) $y = \sin^2 x + \sin^2 x + 1$

$\sin^2 x = 1, 0, \frac{1}{4} \Rightarrow R_f = [1, 3]$

$y = 3, 1, \frac{5}{4}$