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الف)  $y = \frac{|x|+1}{|x|-2}$   $\begin{cases} \max \rightarrow \infty \\ \min \rightarrow -\frac{1}{2} \end{cases} \Rightarrow R_f = (-\infty, -\frac{1}{2}] \cup (1, +\infty)$  -1

صفرية  $x \neq \pm 2$

ب)  $y = \frac{x^2+1}{|x|} \rightarrow |x| + \frac{1}{|x|} \Rightarrow R_f = [1, +\infty)$

$y = \frac{x^2+2x-2}{x+2} \rightarrow 2y \neq y = x^2+2x-2 \Rightarrow x^2+(1-y)x-2-y=0$  12

$\Rightarrow x = \frac{y-2 \pm \sqrt{y^2-4y+4+4y+4}}{2} \Rightarrow y^2+4y+4 \geq 0 \Rightarrow (y+2)(y+2) \geq 0$

$\frac{-4}{4} \pm \frac{-2}{4}$

$\Rightarrow R_f = (-\infty, -2] \cup [-2, +\infty)$

$f(x) = \frac{x^2-x+\frac{1}{x}}{x^2-x+1} = 1 - \frac{\frac{1}{x}}{x^2-x+1}$  12

$x > 0$   $x = \frac{1}{t} \Rightarrow \frac{1}{t} \in [\frac{1}{2}, +\infty) \Rightarrow \frac{1}{t(x^2-x+1)} \rightarrow R_f = (0, 1]$

$\Rightarrow 1 - \frac{1}{t(x^2-x+1)} \rightarrow R_f = (0, 1], R_f = \mathbb{R}$

الف)  $y = \sqrt{-2x^2+5x-1} = \sqrt{(-\infty, 2]} \rightarrow R_f = [0, \sqrt{2}]$  15

$x = \frac{b}{2a} = \frac{5}{4} \Rightarrow y = \sqrt{2}$

ب)  $y = \sqrt[3]{2x^2-11x} \Rightarrow \sqrt[3]{(-1, +\infty)} \Rightarrow R_f = [-2, +\infty)$

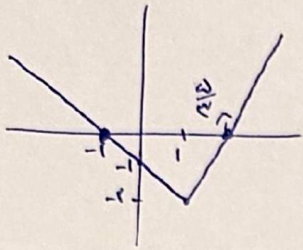
$x = \frac{b}{2a} = \frac{11}{4} \Rightarrow y = -2$

الف)  $y = \log_2(x^2-2x+1) \rightarrow R_f = \mathbb{R} \Rightarrow \log_2(-\infty, +\infty) \rightarrow R_f = \mathbb{R}$  10

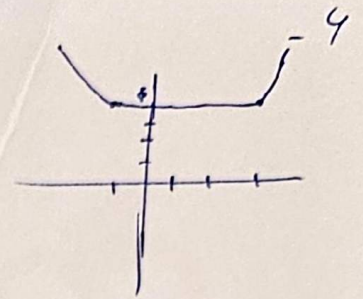
ب)  $\log_2 \frac{x^2+1}{x} \rightarrow \log_2 \frac{x+\frac{1}{x}}{1} \xrightarrow{x>0} \log_2 \frac{x+\frac{1}{x}}{1} \rightarrow R_f = (1, +\infty)$

$x < 0 \Rightarrow \log_2(-\infty, -2] \rightarrow$  غير معرف

الف)  $y = |2x-2| - |2-2| \rightarrow R_f = [-2, +\infty)$

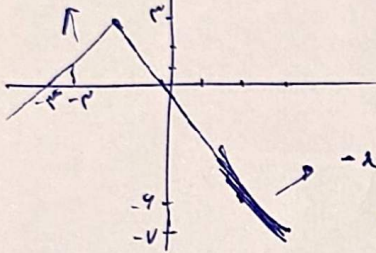


ب)  $y = |2-2| + |2+1|$   
 $R_f = \{1, +\infty\}$



$|2-2| - |2x+2| \leq 1$

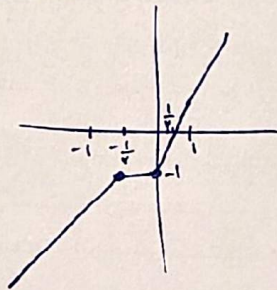
$-2 + 2 + 2 + 2 \leq 1 \Rightarrow x \leq -2$



$\Rightarrow R_f = (-\infty, -2] \cup [-\frac{1}{2}, +\infty)$

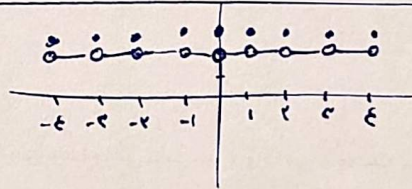
$-2 + 2 - 2 - 2 \leq 1$   
 $\Rightarrow x \geq -\frac{1}{2}$

$2x - |2+1| + |x|$



$\Rightarrow R_f = \mathbb{R}$

ج)  $y = [x] + [x-2] = [x] + [-2] + 2$   
 $\Rightarrow R_f = \{1, 2\}$



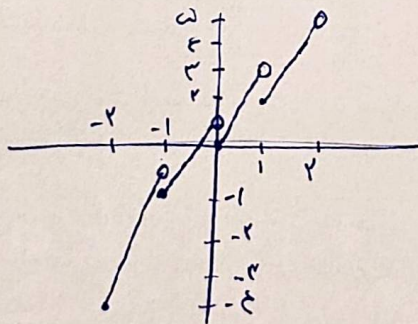
د)  $2x - [x]$

$0 \leq x < 1 \rightarrow 2x$

$1 \leq x < 2 \rightarrow 2x-1$

$-1 \leq x < 0 \rightarrow 2x+1$

$x \leq -1 \rightarrow 2x+2$



$R_f = [-1, +\infty)$

الف)  $y = \sin^2 x - \cos^2 x + 1 \xrightarrow{\text{max}} \frac{1}{2} \rightarrow \frac{1}{2} \Rightarrow R_f = [\frac{1}{2}, 1]$

ب)  $y = \sin^2 x + \sin^2 x + 1 \xrightarrow{\text{max}} 1 \rightarrow 1 \Rightarrow R_f = [1, 2]$