

الف) $x = \frac{|u|+1}{|u|-1} \rightarrow |u| = \frac{xy+1}{y-1} \rightarrow \frac{-\frac{1}{y}}{+\frac{1}{y}-\frac{1}{y}} \rightarrow R_f = (-\infty, -\frac{1}{y}] \cup (1, +\infty)$ (1)

ب) $y = \frac{u^2+1}{|u|} = |u| + \frac{1}{|u|} \rightarrow R_f = [1, +\infty)$

$y = \frac{u^2+1}{u+1} \rightarrow y(u+1) = u^2+1 \rightarrow u^2+(1-y)u-1 = 0 \rightarrow u = \frac{(y-1) \pm \sqrt{(1-y)^2+4(1+y)}}{2}$ (2)

$\rightarrow u = \frac{(y-1) \pm \sqrt{(1-y)^2+4(1+y)}}{2} \rightarrow y^2+1 \geq 0 \rightarrow R_f = (-\infty, -1] \cup [1, +\infty)$

$f(u) = \frac{u^2-u+\frac{1}{x}}{u^2-u+1} \rightarrow y(u^2-u+\frac{1}{x}) = u^2-u+1 \rightarrow (y-1)u^2+(1-y)u+(y-\frac{1}{x}) = 0 \rightarrow u = \frac{(y-1) \pm \sqrt{(1-y)^2-4(y-\frac{1}{x})}}{2y-1}$ (3)

$\rightarrow \begin{cases} -4y^2+4y > 0 \\ y \neq 1 \end{cases} \rightarrow 0 < y < 1 \rightarrow R_f = (0, 1) \quad R_f = \mathbb{R}$

الف) $y = \sqrt{-u^2+1} \rightarrow y_{\max} = 1 \quad R_f = [0, 1]$ (4)

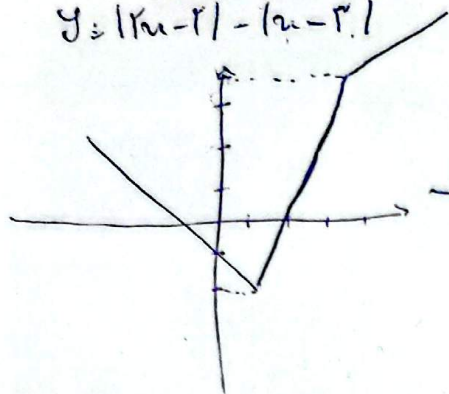
ب) $y = \sqrt{u^2-1} \rightarrow y_{\min} = -1 \rightarrow R_f = [-1, +\infty)$

الف) $y = \log(u^2-u+1) \rightarrow R = \mathbb{R} \xrightarrow{1 \leq y} R_f = \mathbb{R}$ (5)

ب) $y = \log_r(\frac{u^2+1}{u}) \rightarrow R(\frac{u^2+1}{u}) = (-\infty, -1] \cup [1, +\infty) \xrightarrow{1 \leq y} R_f = [1, +\infty)$

$$y = |ru - r| - |u - r|$$

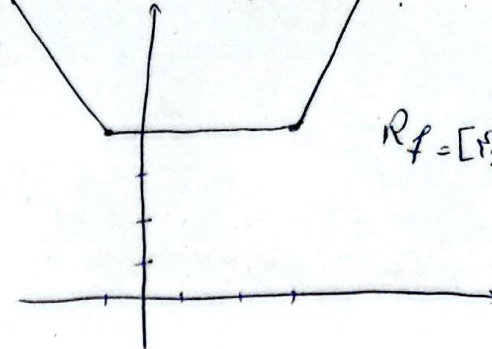
(الف)



$$\rightarrow R_f = [-r, +\infty)$$

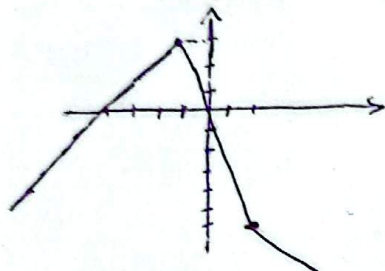
$$y = |u - r| + |u + r|$$

(ب)



$$R_f = [r, +\infty)$$

$$|u - r| - |u + r| \leq 1 \rightarrow u \in [r, -\frac{1}{r})$$



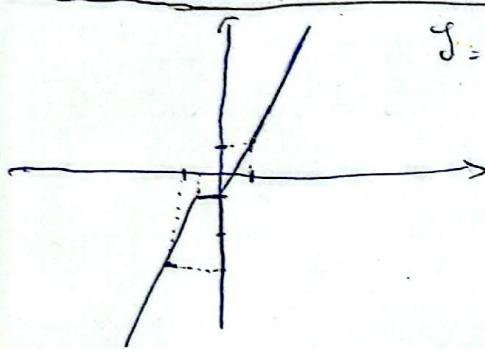
$$\begin{aligned} u + r < 1 & \quad | -ru + r | - u - r \leq 1 \\ u &\leq -r \quad u \geq -\frac{1}{r} \quad u \geq 0 \end{aligned}$$

$$\rightarrow R_f = (-r, -\frac{1}{r})$$

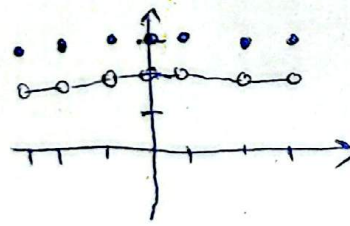
$$y = ru - |ru + 1| + |u|$$

$$\begin{array}{c|c|c} -\frac{1}{r} & 0 & \\ \hline ru+1 & -1 & ru-1 \end{array}$$

$$\rightarrow R_f = \mathbb{R}$$

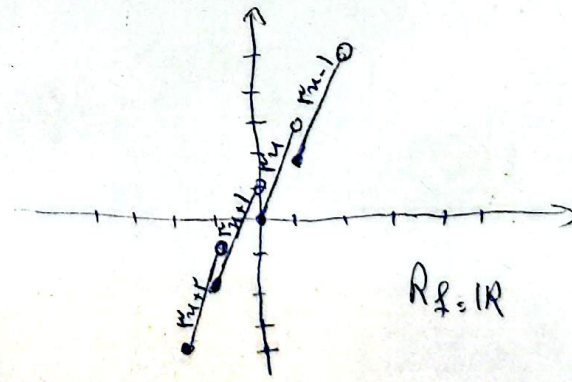


$$\text{ج) } y = [u] + [r - u] = [u] + [-u] + r$$



$$\text{د) } y = ru - [u]$$

(17)



$$R_f = \mathbb{R}$$

$$\text{الف) } y = f_{\sin}^r - f_{\sin}^{u+1} \rightarrow \begin{cases} f_{\sin}^u = 1 \rightarrow y = 1 \\ f_{\sin}^u = -1 \rightarrow y = r \\ f_{\sin}^u = \frac{1}{r} \rightarrow y = \frac{r}{r} \end{cases}$$

$$\rightarrow R_f = [\frac{r}{r}, r]$$

(1)

$$\text{ب) } y = f_{\sin}^f + f_{\sin}^{r+1}$$

$$\begin{cases} f_{\sin}^u = 0 \rightarrow y = 1 \\ f_{\sin}^u = 1 \rightarrow y = r \\ f_{\sin}^u = -\frac{1}{r} \end{cases} \rightarrow R_f = [1, r]$$