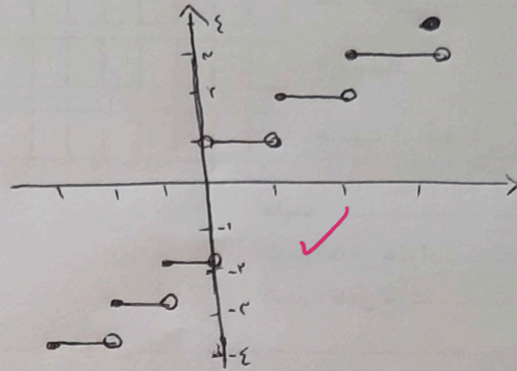


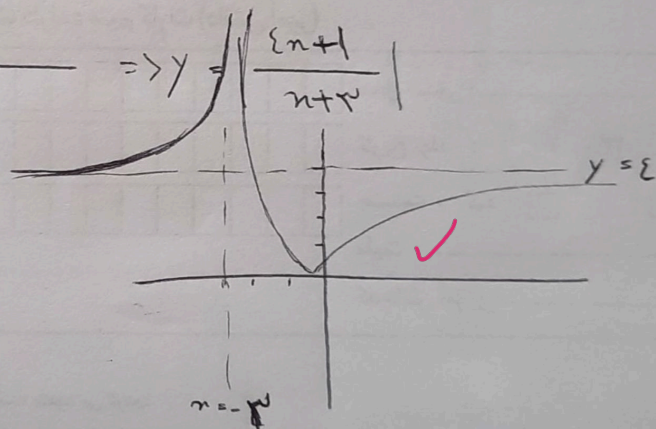
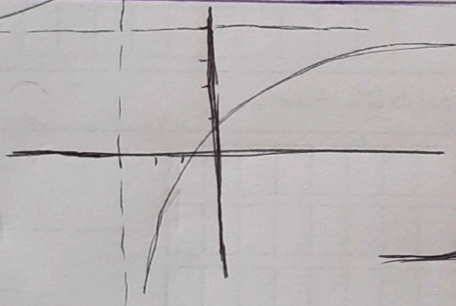
الف) $-1 \leq \sin(n + \frac{\pi}{2}) \leq 1 \Rightarrow 0 \leq \sin^2(n + \frac{\pi}{2}) \leq 1 \Rightarrow 0 \leq 2\sin^2(n + \frac{\pi}{2}) \leq 2$
 $\Rightarrow -1 \leq 2\sin^2(n + \frac{\pi}{2}) \leq 1 \Rightarrow R_f = [-1, 1]$

ب) $\cos n = -1 \rightarrow 2\cos^2 n - 3 = 2(-1)^2 - 3 = -1$
 $\cos n = 1 \rightarrow 2\cos^2 n - 3 = 2(1)^2 - 3 = -1 \Rightarrow R_f = [-1, 1]$

$f(n) = \begin{cases} -1-2 = -3 & -2 \leq n < -1 \\ -1-2 = -3 & -1 \leq n < 0 \\ -1-1 = -2 & 0 \leq n < 1 \\ 1 & 1 \leq n < 2 \\ 1+1 = 2 & 2 \leq n < 3 \\ 1+2 = 3 & n = 3 \end{cases}$



$y = \frac{\varepsilon n + 1}{n + 2}$

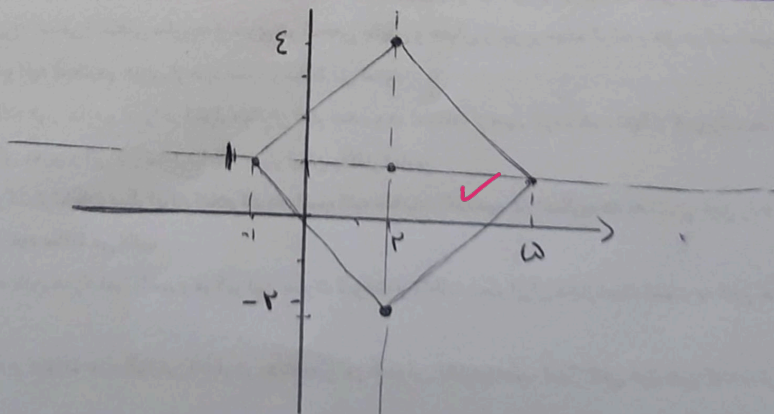


$\frac{n}{y} \geq 2 \rightarrow y = \varepsilon - n$

$\frac{n}{y} < 2 \rightarrow y = n + \varepsilon$

$\frac{n}{y} \geq 2 \rightarrow y = n - \varepsilon$

$\frac{n}{y} < 2 \rightarrow y = n$



ا) $\sqrt{n + \frac{1}{n}} = \square \Rightarrow \square^2 + 2\square = n + \frac{1}{n} \Rightarrow \square^2 + 2\square - n - \frac{1}{n} = 0$

$\square = \sqrt{n + \frac{1}{n}} \Rightarrow f(n) = \sqrt{n + \frac{1}{n}}$
 $\square = -\sqrt{n + \frac{1}{n}} \Rightarrow f(n) = -\sqrt{n + \frac{1}{n}}$

$\square = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = -1 \pm \sqrt{1 + 4n} \Rightarrow R_f = [-1 + \sqrt{1 + 4n}, +\infty)$

ب) $(2 - \sin n)(1 + \sin n) = -\sin^2 n + 2\sin n + 2 = -(\sin n - 1)^2 + 3$
 $\Rightarrow R_f = [0, 3]$

$\hookrightarrow \sin n = 1 \Rightarrow f(n) = 3$
 $\hookrightarrow \sin n = -1 \Rightarrow f(n) = 0$

$$r\mathbb{Q} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \Rightarrow R_f = \left\{ \frac{r_1}{r_2}, \frac{r_1}{r_3}, \frac{r_1}{r_4}, \frac{r_1}{r_5}, \frac{r_1}{r_6}, \frac{r_1}{r_7}, \frac{r_1}{r_8}, \frac{r_1}{r_9} \right\}$$

$$\Rightarrow \left(\frac{r_1}{r_2} \right) = \frac{r_1}{r_2} = \boxed{V_1}$$

$$a=0 \Rightarrow f(n) = \sqrt{bn+c} \xrightarrow{n=r} \sqrt{rb+c} = 0 \Rightarrow rb+c=0$$

$$\xrightarrow{n=r} \sqrt{rb+c} = 1 \Rightarrow rb+c=1 \Rightarrow \begin{matrix} b=1 \\ c=r \\ a=0 \end{matrix}$$

$$\Rightarrow g(n) = \sqrt{n^2 + \varepsilon} \Rightarrow \varepsilon \leq n^2 + \varepsilon < +\infty \Rightarrow R_{(g)} = [r, +\infty)$$

$$f(n) = \frac{n(n-r)(n+r)}{n+r} = n(n-r) = n^2 - rn \xrightarrow{\text{div}} n^2 - rn \neq 1 \Rightarrow n^2 - rn - 1 \neq 0$$

$$\Rightarrow n \neq \varepsilon$$

$$n \neq -r \rightarrow \begin{matrix} a=\varepsilon \\ b=-r \end{matrix}$$

$$-1 \leq n^2 - rn < +\infty \Rightarrow c = -1$$

$$\Rightarrow f\left(\frac{a+b}{rc}\right) = f\left(\frac{\varepsilon-r}{-r}\right) = f(-1) = -1(-1-r) = r$$

$$n^2 + an^2 + bn - 1 \rightsquigarrow n^2 - rn^2 + \varepsilon n - 1 = (n-r)^2 \Rightarrow y = n-r$$

$$\xrightarrow{n=-r} y = -1$$

$$\xrightarrow{n=r} x = 1_0 \Rightarrow R_f = [-1, 1_0]$$

$$\begin{matrix} a = -r \\ b = 1r \end{matrix}$$

$$f(n) = \begin{cases} 0+1=1 & \frac{1}{r} \leq n < 1 \\ 1+1=2 & 1 \leq n < \frac{r}{r} \\ 1+r=r & \frac{r}{r} \leq n < r \end{cases}$$

