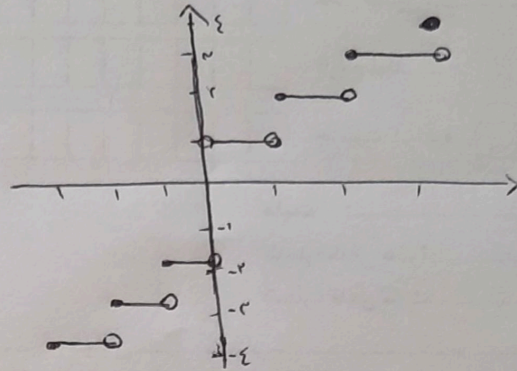


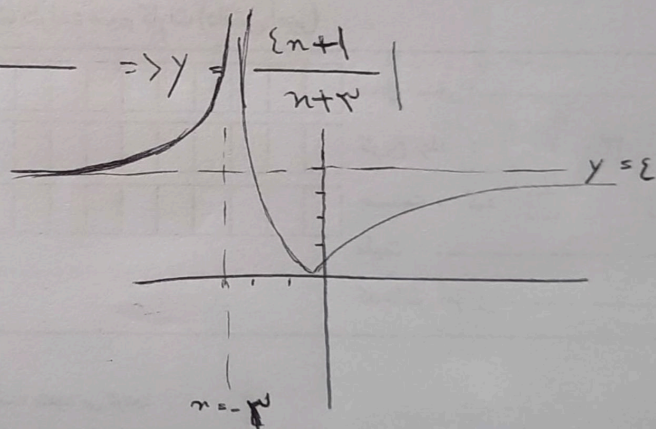
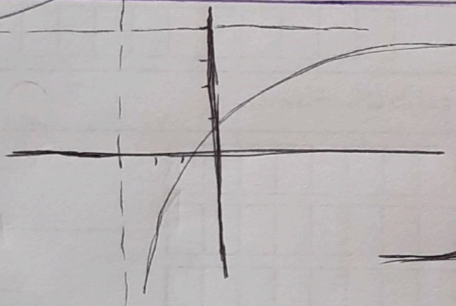
الف) $-1 \leq \sin(n + \frac{\pi}{2}) \leq 1 \Rightarrow 0 \leq \sin^2(n + \frac{\pi}{2}) \leq 1 \Rightarrow 0 \leq 2\sin^2(n + \frac{\pi}{2}) \leq 2$
 $\Rightarrow -1 \leq 2\sin^2(n + \frac{\pi}{2}) \leq 1 \Rightarrow R_f = [-1, 1]$

ب) $\cos n = -1 \rightarrow 2\cos^2 n - 1 = 2(-1)^2 - 1 = 1$
 $\cos n = 1 \rightarrow 2\cos^2 n - 1 = 2(1)^2 - 1 = 1 \Rightarrow R_f = [-1, 1]$

$f(n) = \begin{cases} -1-2 = -3 & -3 \leq n < -2 \\ -1-2 = -3 & -2 \leq n < -1 \\ -1-1 = -2 & -1 \leq n < 0 \\ 1 & 0 \leq n < 1 \\ 1+1 = 2 & 1 \leq n < 2 \\ 1+2 = 3 & 2 \leq n < 3 \\ \epsilon & n = 3 \end{cases}$



$y = \frac{\epsilon n + 1}{n + 2}$

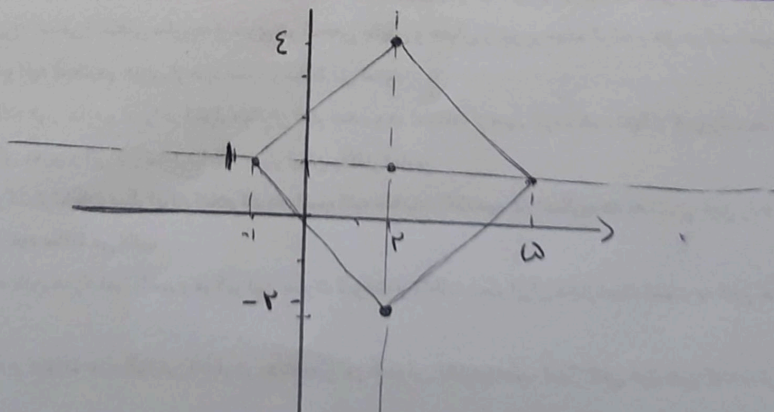


$\frac{n}{y} > 2 \rightarrow y = 9 - n$
 $y \geq 1$

$\frac{n}{y} < 2 \rightarrow y = n + 2$
 $y \geq 1$

$\frac{n}{y} > 2 \rightarrow y = n - \epsilon$
 $y < 1$

$\frac{n}{y} < 2 \rightarrow y = n$
 $y < 1$



الف) $\sqrt{n + \frac{1}{n}} = \square \Rightarrow \square^2 = n + \frac{1}{n} \Rightarrow \square^2 - n = \frac{1}{n} \Rightarrow \square^2 = n + \frac{1}{n}$
 $\square = \sqrt{n + \frac{1}{n}} \Rightarrow f(n) = \sqrt{n + \frac{1}{n}}$
 $\square = +\infty \Rightarrow f(n) = +\infty$

$\square = \frac{a-b}{\sqrt{a^2+b^2}} = -1 \Rightarrow R_f = [\sqrt{n + \frac{1}{n}}, +\infty]$

ب) $(2 - \sin n)(1 + \sin n) = -\sin^2 n + 2\sin n + 2 = -(\sin n - 1)^2 + 3$
 $\Rightarrow R_f = [0, 3]$

$\hookrightarrow \sin n = 1 \Rightarrow f(n) = 3$
 $\hookrightarrow \sin n = -1 \Rightarrow f(n) = 0$

$$r\mathbb{Z} = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \Rightarrow R_f = \left\{ \frac{r_1}{r}, \frac{r_2}{r}, \frac{r_3}{r}, \frac{r_4}{r}, \frac{r_5}{r}, \frac{r_6}{r}, \frac{r_7}{r}, \frac{r_8}{r}, \frac{r_9}{r} \right\}$$

$$\Rightarrow \left(\frac{r_1}{r} \times \frac{r_5}{r} \right) = \frac{r_1 r_5}{r \times r} = \boxed{r_5}$$

$$a=0 \Rightarrow f(n) = \sqrt{bn+c} \xrightarrow{n=r} \sqrt{rb+c} = 0 \Rightarrow rb+c=0$$

$$\xrightarrow{n=r} \sqrt{rb+c} = 1 \Rightarrow rb+c=1 \Rightarrow \begin{cases} b=1 \\ c=-r \\ a=0 \end{cases}$$

$$\Rightarrow g(n) = \sqrt{n^2 + \varepsilon} \Rightarrow \varepsilon \leq n^2 + \varepsilon < +\infty \Rightarrow R_{(g)} = \boxed{[r, +\infty)}$$

$$f(n) = \frac{n(n-r)(n+r)}{n+r} = n(n-r) = n^2 - rn \xrightarrow{\text{div}} n^2 - rn \neq 1 \Rightarrow n^2 - rn - 1 \neq 0$$

$$\Rightarrow \begin{cases} n \neq \varepsilon \\ n \neq -r \end{cases} \rightarrow \begin{cases} a = \varepsilon \\ b = -r \end{cases}$$

$$-1 \leq n^2 - rn < +\infty \Rightarrow \underline{c = -1}$$

$$\Rightarrow f\left(\frac{a+b}{rc}\right) = f\left(\frac{\varepsilon-r}{-r}\right) = f(-1) = -1(-1-r) = r$$

$$n^2 + an^2 + bn - 1 \rightsquigarrow n^2 - rn^2 + \varepsilon n - 1 = (n-r)^2 \Rightarrow y = n-r$$

$$\xrightarrow{n=-r} y = -1$$

$$\xrightarrow{n=r} x = 1_0 \Rightarrow R_f = \boxed{[-1, 1_0]}$$

$$\begin{cases} a = -r \\ b = 1r \end{cases}$$

$$f(n) = \begin{cases} 0+1=1 & \frac{1}{r} \leq n < 1 \\ 1+1=2 & 1 \leq n < \frac{r}{r} \\ 1+r=r & \frac{r}{r} \leq n < r \end{cases}$$

