

الف) $0 \leq \sin^2\left(x + \frac{\pi}{3}\right) \leq 1 \rightarrow 0 \leq \sin^2\left(x + \frac{\pi}{3}\right) \leq 1 \rightarrow -1 \leq \sin\left(x + \frac{\pi}{3}\right) \leq 1$
 $\Rightarrow R_{f(x)} = [-1, 1]$

ب) $-1 \leq \cos^2 x \leq 1 \rightarrow -1 \leq \cos^2 x \leq 1 \rightarrow -1 \leq \cos x \leq 1$
 $\Rightarrow R_{g(x)} = [-1, 1]$

$x > 0$
 $\rightarrow 1 + [x] = \begin{cases} 0 < x < 1 \rightarrow 1 + [x] = 1 \\ 1 < x < 2 \rightarrow 1 + [x] = 2 \\ 2 < x < 3 \rightarrow 1 + [x] = 3 \end{cases}$

$x < 0$
 $\rightarrow -1 + [x] = \begin{cases} -1 < x < 0 \rightarrow -1 + [x] = -1 \\ -2 < x < -1 \rightarrow -1 + [x] = -2 \\ -3 < x < -2 \rightarrow -1 + [x] = -3 \end{cases}$

$y = \frac{x+1}{x+3}$

$|y-1| = 3 - |x-2|$

$\rightarrow y \geq 1, x \geq 2 \Rightarrow y-1 = 3-x+2 \Rightarrow y = -x+6$
 $\rightarrow y \geq 1, x < 2 \Rightarrow y-1 = 3+x-2 \Rightarrow y = x+2$
 $\rightarrow y < 1, x \geq 2 \Rightarrow 1-y = 3-x+2 \Rightarrow y = x-4$
 $\rightarrow y < 1, x < 2 \Rightarrow 1-y = 3+x-2 \Rightarrow y = -x$

الف) $x + \frac{1}{x} \geq 2 \Leftrightarrow x + \frac{1}{x} \leq -2 \xrightarrow{\text{با } x \text{ ضرب}} x + \frac{1}{x} \geq 2 \Rightarrow x + \frac{1}{x} + 2 \sqrt{x + \frac{1}{x}}$
 $\rightarrow R_{f(x)} = [2 + 2\sqrt{2}, +\infty)$

ب) $(3 - \sin x)(1 + \sin x) = -\sin^2 x + 2\sin x + 3$
 $\Rightarrow R_{g(x)} = [-1, 3]$

$$D = \{1, 2, 3, \dots, 9\} \Rightarrow -\frac{1}{p} + 1_0 - \frac{2}{p} + 1_0 - \frac{3}{p} + 1_0 + \dots - \frac{9}{p} + 1_0$$

$$= -\frac{1+\dots+9}{p} + 9 \times 1_0 = -\frac{45}{p} + 9_0 = 9_0 - 1_0 = \sqrt{81}$$

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تابع خطی است $\Rightarrow a=0 \Rightarrow \sqrt{b}x+c$ $\begin{matrix} x=2 \\ y=0 \end{matrix} \Rightarrow 2b+c=0$, $\begin{matrix} x=3 \\ y=1 \end{matrix} \Rightarrow 3b+c=1$

$$\Rightarrow b=1, c=-2$$

$$\Rightarrow f(x) = \sqrt{2x^2+5} \xrightarrow{\text{محدود}} \sqrt{[4, +\infty)} \Rightarrow [2, +\infty) \Rightarrow R_f: [2, +\infty)$$

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$$f(m) = \frac{a_1(m-2)(m+2)}{a_1+2} \xrightarrow{a_1 \neq -2} f(m) = a_1(m-2), f(m) \neq 1$$

$$\Rightarrow a_1^2 - 2a_1 \neq 1 \Rightarrow a_1^2 - 2a_1 - 1 \neq 0 \quad \begin{matrix} a_1 \neq -2 \\ a_1 \neq 2 \end{matrix} \Rightarrow R_{f(m)} = \mathbb{R} - \{-2, 2\}$$

$$f(m) = a_1^2 - 2a_1 \xrightarrow{a_1} a_1, \frac{y}{x} = 1, y_1 = -1 \Rightarrow R_{f(m)} = [-1, +\infty) - \{1\} \Rightarrow \begin{matrix} a = -2 \\ b = 2 \\ c = -1 \end{matrix}$$

$$f\left(\frac{a+b}{2}\right) = f(-1) = \sqrt{3}$$

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خطی $y = \sqrt{(x-2)^2} = \sqrt{x^2 - 4x + 4} = x - 2$, $\begin{matrix} a = -9 \\ b = 15 \end{matrix}$

$$\Rightarrow R_f: [-9, 15] \Rightarrow -9 \leq x \leq 15 \sim -1 \leq x-2 \leq 10$$

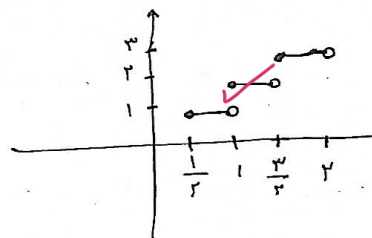
$$\Rightarrow R_f: [-1, 10]$$

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$$\frac{1}{2} \leq m < 1 \quad [m] = 0, [m + \frac{1}{2}] = 1 \Rightarrow [m] + [m + \frac{1}{2}] = 1$$

$$1 \leq m < \frac{3}{2} \quad [m] = 1, [m + \frac{1}{2}] = 1 \Rightarrow [m] + [m + \frac{1}{2}] = 2$$

$$\frac{3}{2} \leq m < 2 \quad [m] = 1, [m + \frac{1}{2}] = 2 \Rightarrow [m] + [m + \frac{1}{2}] = 3$$



$$\Rightarrow \text{تابع پله‌ای است}$$

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