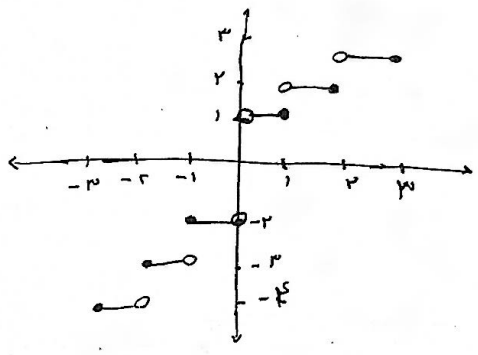


الف)  $0 \leq \sin^2\left(a + \frac{\pi}{3}\right) \leq 1 \rightarrow 0 \leq \sin^2\left(a + \frac{\pi}{3}\right) \leq 1 \rightarrow -1 \leq \sin\left(a + \frac{\pi}{3}\right) \leq 1$   
 $\rightarrow -1 \leq \sin\left(a + \frac{\pi}{3}\right) \leq 1 \Rightarrow R_{\sin} = [-1, 1]$

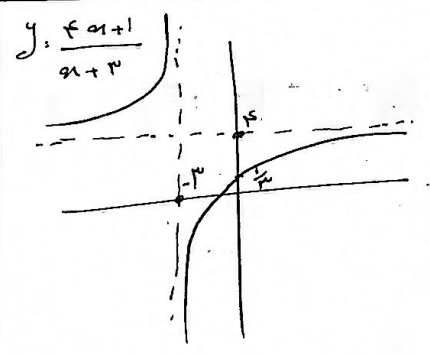
ب)  $-1 \leq \cos^2 a \leq 1 \rightarrow -1 \leq \cos^2 a \leq 1 \rightarrow -1 \leq \cos a \leq 1$   
 $\Rightarrow R_{\cos} = [-1, 1]$

$a > 0$   
 $\rightarrow 1 + [a] = \begin{cases} 0 < a < 1 \rightarrow 1 + [a] = 1 \\ 1 < a < 2 \rightarrow 1 + [a] = 2 \\ 2 < a < 3 \rightarrow 1 + [a] = 3 \end{cases}$

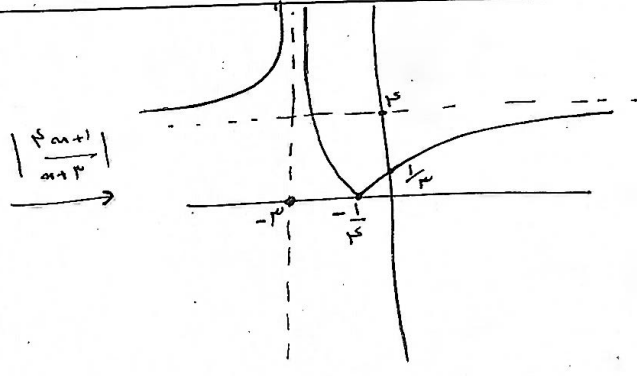
$a < 0$   
 $\rightarrow -1 + [a] = \begin{cases} -1 < a < 0 \rightarrow -1 + [a] = -1 \\ -2 < a < -1 \rightarrow -1 + [a] = -2 \\ -3 < a < -2 \rightarrow -1 + [a] = -3 \end{cases}$



$y = \frac{4a+1}{a+3}$

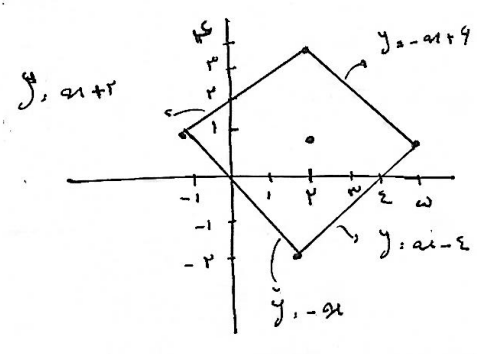


$\left| \frac{4a+1}{a+3} \right|$



$|y-1| = 3 - |a-2|$

$\sim y \geq 1, a \geq 2 \Rightarrow y-1 = 3-a+2 \Rightarrow y = -a+6$   
 $\sim y \geq 1, a < 2 \Rightarrow y-1 = 3+a-2 \Rightarrow y = a+2$   
 $\sim y < 1, a \geq 2 \Rightarrow 1-y = 3-a+2 \Rightarrow y = a-2$   
 $\sim y < 1, a < 2 \Rightarrow 1-y = 3+a-2 \Rightarrow y = -a$



الف)  $a + \frac{1}{a} \geq 2 \quad \text{و} \quad a + \frac{1}{a} \leq -2 \quad \xrightarrow{\text{با } \frac{a}{a}}$   $a + \frac{1}{a} \geq 2 \Rightarrow a + \frac{1}{a} + 2\sqrt{a + \frac{1}{a}}$

$\rightarrow R_{\sin} = [2+2\sqrt{2}, +\infty)$

ب)  $(3 - \sin a)(1 + \sin a) = -\sin^2 a + 2\sin a + 3$

$\begin{aligned} \sin a = 1 &\rightarrow -1 + 2 + 3 = 4 \\ \sin a = -1 &\rightarrow -1 - 2 + 3 = 0 \\ \sin a = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-2 \pm \sqrt{4 - 12}}{2} = 1 \end{aligned} \Rightarrow R_y = [0, 4]$

$$D = \{1, 2, 3, \dots, 9\} \Rightarrow -\frac{1}{1} + 1_0 - \frac{2}{2} + 1_0 - \frac{3}{3} + 1_0 + \dots - \frac{9}{9} + 1_0$$

$$= -\frac{1+\dots+9}{9} + 9 \times 1_0 = -\frac{45}{9} + 9_0 = 9_0 - 1_0 = \boxed{8}$$

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تابع خطی است  $\rightarrow a=0 \Rightarrow \sqrt{bx+c}$   $\xrightarrow{x=2, y=0} 2b+c=0, 3b+c=1$

$$\Rightarrow b=1, c=-2$$

$$\Rightarrow f(x) = \sqrt{2x^2+4} \xrightarrow{\text{محدود}} \sqrt{[4, +\infty)} = [2, +\infty) \Rightarrow R_f: [2, +\infty)$$

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$$f(x) = \frac{x(x-2)(x+2)}{x+2} \xrightarrow{x \neq -2} f(x) = x(x-2), f(x) \neq 1$$

$$\Rightarrow x^2 - 2x \neq 1 \Rightarrow x^2 - 2x - 1 \neq 0 \quad \begin{cases} x \neq -2 \\ x \neq 2 \end{cases} \Rightarrow D_{f(x)} = \mathbb{R} - \{-2, 2\}$$

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$$f(x) = x^2 - 2x \rightarrow 2b, \frac{1}{2} = 1, y_0 = -1 \Rightarrow R_{f(x)} = [-1, +\infty) - \{1\} \Rightarrow \begin{cases} a = -2 \\ b = 1 \\ c = -1 \end{cases}$$

$$f\left(\frac{a+b}{2}\right) = f(-1) = \boxed{3}$$

تابع  $y = \sqrt[3]{(x-2)^3} = \sqrt[3]{x^3 - 6x^2 + 12x - 8} = x - 2, a = -9, b = 12$

$$\Rightarrow R_f: [-9, 12] \Rightarrow -9 \leq x \leq 12 \sim -1 \leq x-2 \leq 10$$

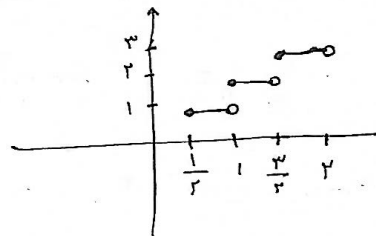
9

$$\Rightarrow R_f: [-1, 10]$$

$$\frac{1}{2} \leq x < 1 \quad [x] = 0, [x + \frac{1}{2}] = 1 \Rightarrow [x] + [x + \frac{1}{2}] = 1$$

$$1 \leq x < \frac{3}{2} \quad [x] = 1, [x + \frac{1}{2}] = 1 \Rightarrow [x] + [x + \frac{1}{2}] = 2$$

$$\frac{3}{2} \leq x < 2 \quad [x] = 1, [x + \frac{1}{2}] = 2 \Rightarrow [x] + [x + \frac{1}{2}] = 3$$



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$$\Rightarrow \text{از } 3 \text{ به } 0 \text{ متناقص است}$$