

الف) $f(n) = 2 \sin^2(n + \frac{\pi}{4}) - 1$

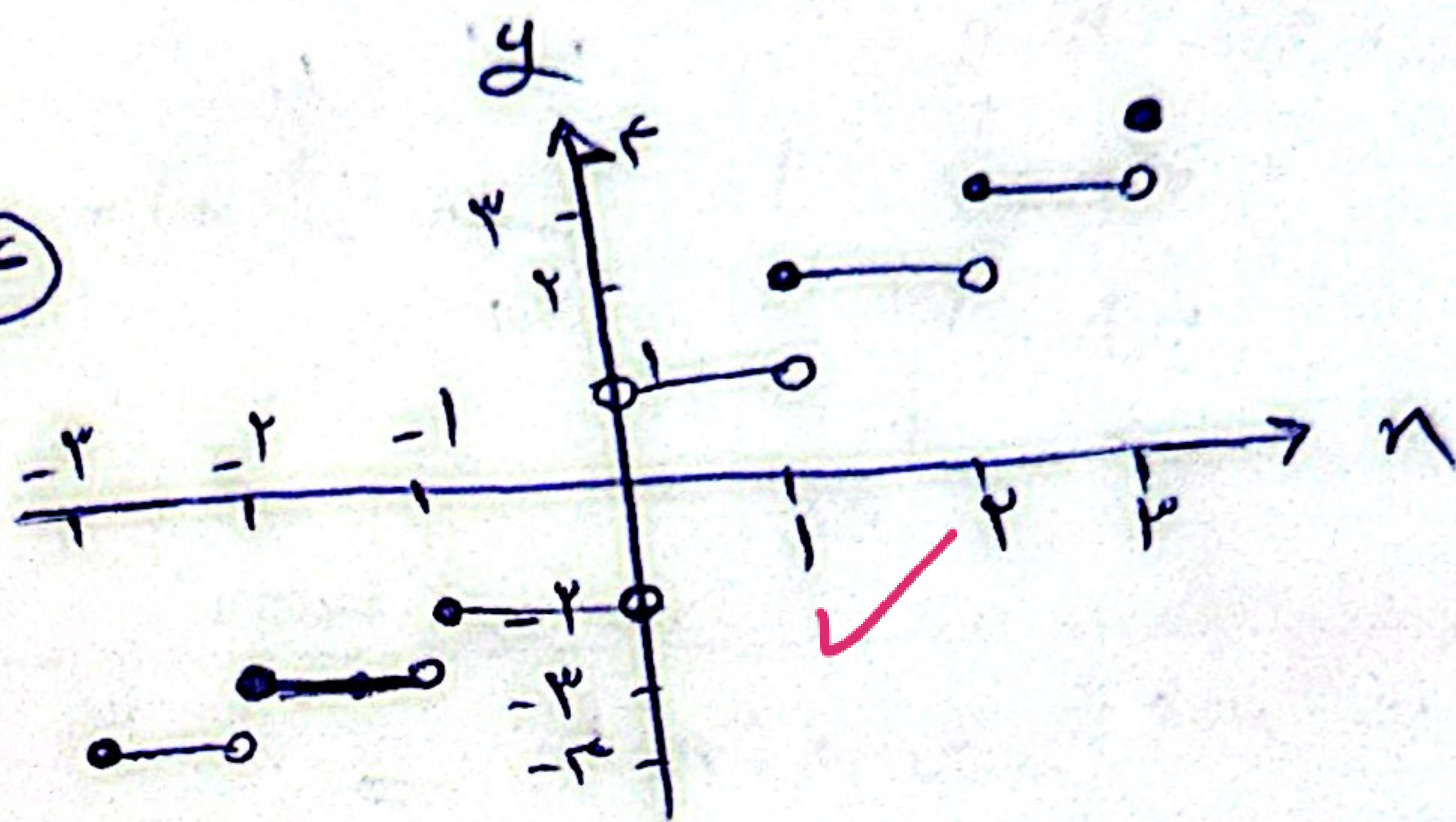
$0 \leq \sin^2 \omega \leq 1 \rightarrow -1 \leq 2 \sin^2(n + \frac{\pi}{4}) - 1 \leq 1 \rightarrow R_f = [-1, 1]$

ب) $g(n) = 2 \cos^3 n - 2$

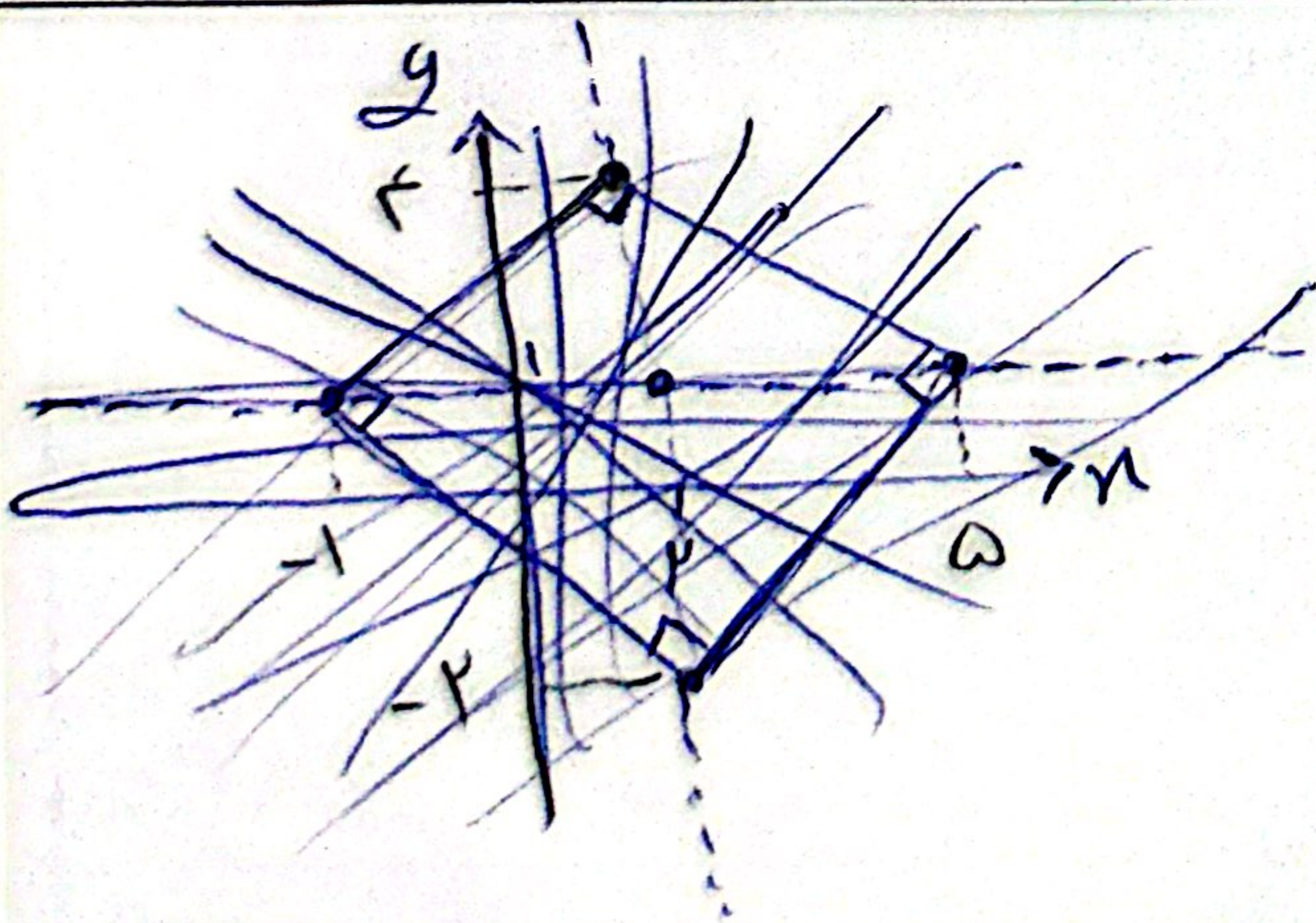
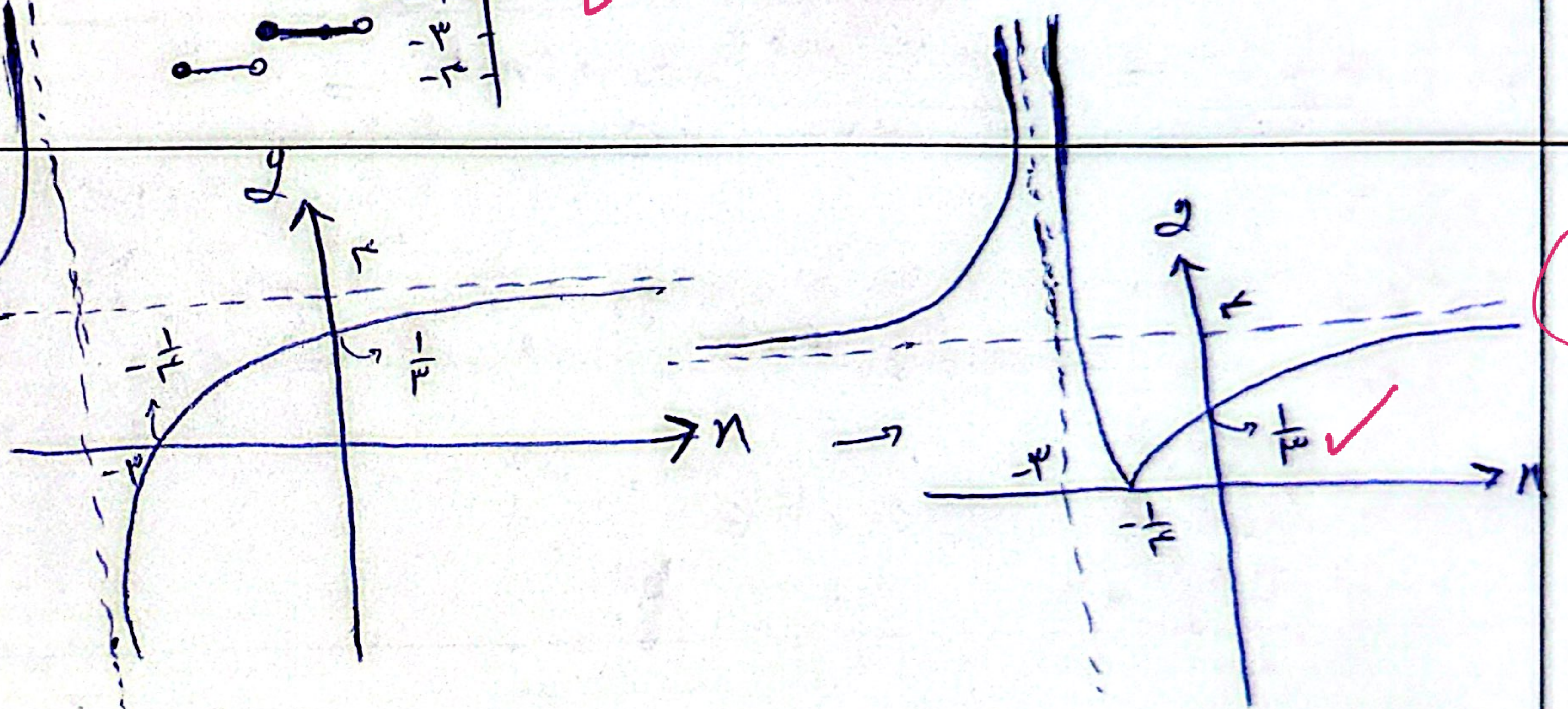
$-1 \leq \cos n \leq 1 \rightarrow -2 \leq 2 \cos^3 n - 2 \leq -1 \rightarrow R_g = [-2, -1]$

$g = \frac{n}{|n|} + [n] \rightarrow [-3, 3]$

$n = 3 \rightarrow (3)$
 $-3 \leq n < -2 \rightarrow (-4)$
 $-2 \leq n < -1 \rightarrow (-2)$
 $-1 \leq n < 0 \rightarrow (-1)$
 $0 \leq n < 1 \rightarrow (1)$
 $1 \leq n < 2 \rightarrow (2)$
 $2 \leq n < 3 \rightarrow (3)$

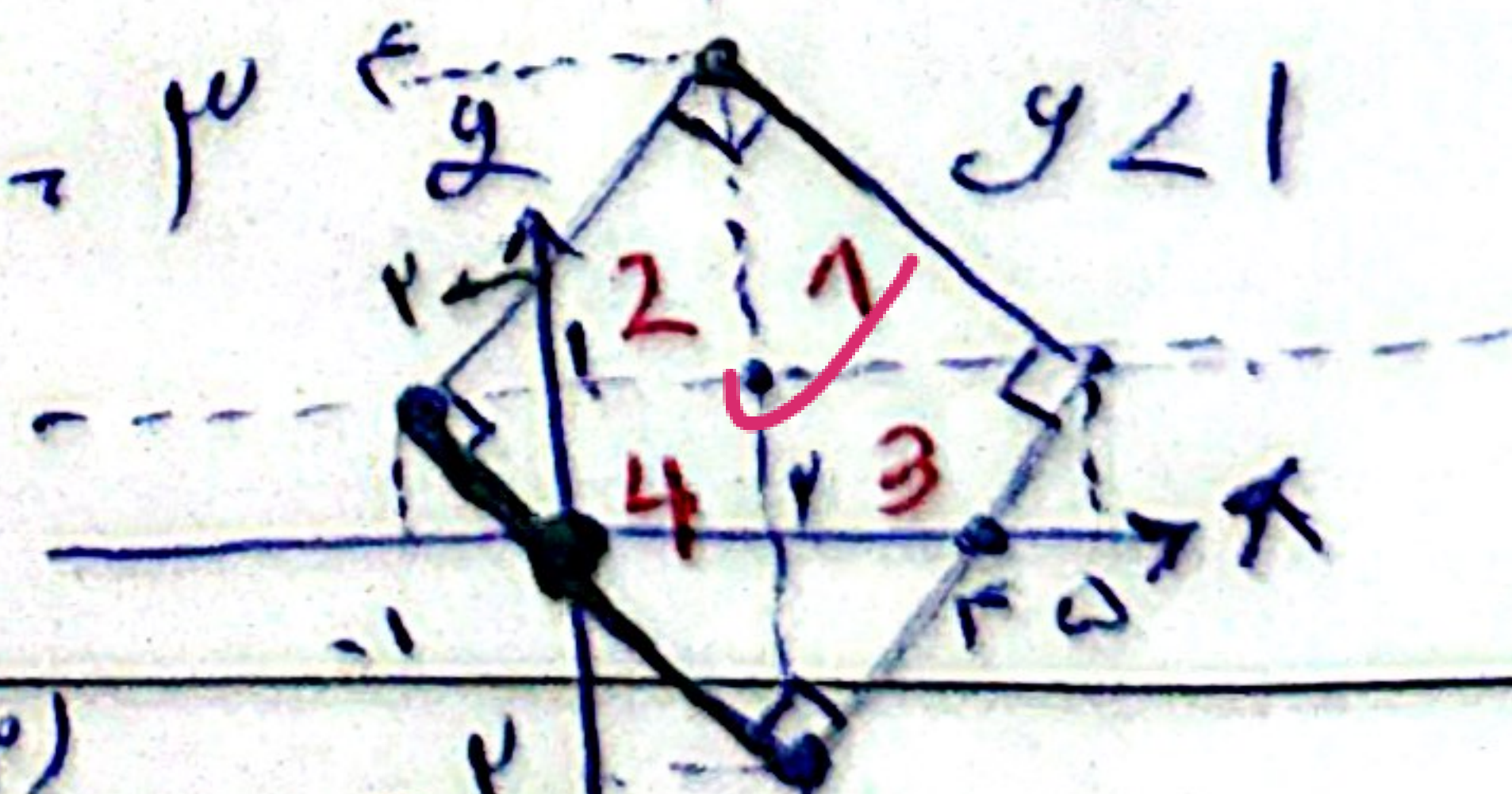


$g = \left| \frac{n+1}{n+3} \right|$



$n - 2 + g - 1 \geq 3$
 $g \geq n + 4$
 $-n + 2 + g - 1 \geq 3$
 $g \geq n + 2$
 $n - 2 - g + 1 \geq 3$
 $n - 4 \geq g$
 $-n + 2 - g + 1 \geq 3$
 $-n \geq g$

$g \geq 1 \rightarrow n \geq 2(1)$
 $g \geq 1 \rightarrow n \leq 2(2)$
 $g \leq 1 \rightarrow n \geq 2(3)$
 $g \leq 1 \rightarrow n \leq 2(4)$



الف) $f(n) = n + \frac{1}{n} + 2\sqrt{n + \frac{1}{n}}$

$= (\sqrt{n + \frac{1}{n}} + 1)^2 - 1$

$n + \frac{1}{n} \geq 2 \rightarrow (\sqrt{n + \frac{1}{n}} + 1)^2 - 1 \geq 4$

$(\sqrt{2} + 1)^2 \leq g \leq +\infty$

$2 + 2\sqrt{2} \leq (\sqrt{n + \frac{1}{n}} + 1)^2 - 1 \leq +\infty \rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $g = \frac{-(\sin n - 2)}{(2 - \sin n)(1 + \sin n)}$

$g = \frac{-\sin^2 n + 2 \sin n + 2}{-1 = \min}$

$R = [0, 4]$

$$D_f = \{1, 2, \dots, 9\}$$

$$f(n) = -\frac{1}{p}n + 10$$

$$n=1 \rightarrow y = -\frac{1}{p} + 10$$

$$n=2 \rightarrow y = -\frac{2}{p} + 10$$

$$n=9 \rightarrow y = -\frac{9}{p} + 10$$

$$\left\{ \begin{array}{l} \frac{9-1}{p} \\ -\frac{(1+2+\dots+9)}{p} + 90 = 90 - 10 = \underline{80} \end{array} \right.$$

$$f(n) = \sqrt{an^2 + bn + c}$$

$$D_f = [1, +\infty) \rightarrow a=0 \rightarrow -\frac{c}{b} = 1$$

$$f(1) = \sqrt{a+b+c} = 1 \rightarrow a+b+c=1$$

$$\downarrow$$

$$a+b+c=1$$

$$\downarrow$$

$$b=1$$

$$\downarrow$$

$$c=0$$

$$g(n) = \sqrt{\frac{n^2}{[1, +\infty)} + 1} = \sqrt{[1, +\infty)} = [1, +\infty)$$

$$R_g = [1, +\infty)$$

$$f(n) = \frac{n^3 - 1}{n+1} = \frac{n(n-1)(n+1)}{n+1} \Rightarrow n(n-1) = n^2 - n$$

$$n^2 - n \neq 1$$

$$\downarrow$$

$$n^2 - n - 1 \neq 0$$

$$\downarrow$$

$$(n-1)(n+1) \neq 0$$

$$\downarrow$$

$$n \neq 1$$

$$n \neq -1$$

$$D_f = \mathbb{R} - \{1, -1\}$$

$$R_f = [-1, +\infty) - \{1\}$$

$$\rightarrow f\left(\frac{1}{-1}\right) = f(-1) = \underline{0}$$

$$a, b \in \{-1, 1\}$$

$$\rightarrow a=1, b=-1$$

$$y = \sqrt[n]{n^3 + an^2 + bn - 1} \rightarrow y = \sqrt[n]{n^3 - 4n^2 + 11n - 1}$$

$$\downarrow$$

$$y = n-1$$

$$D = [-4, 11]$$

$$-4 \leq n \leq 11$$

$$-1 \leq n-1 \leq 10 \rightarrow R = [-1, 10]$$

$$y = [n] + [n + \frac{1}{p}]$$

$$n \in [\frac{1}{p}, 1) \rightarrow y = 1$$

$$1 \leq n < \frac{p}{p} \rightarrow y = 2$$

$$\frac{p}{p} \leq n < 2 \rightarrow y = 3$$

