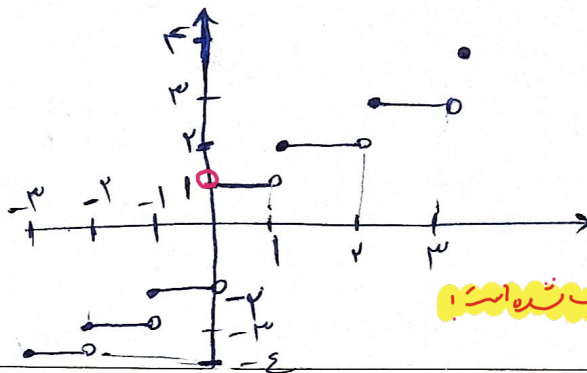


الف) $f(x) = 2 \sin^2(x + \frac{\pi}{3}) - 1 \rightarrow 2[0, 1] - 1 = [0, 2] - 1 = [-1, 1]$

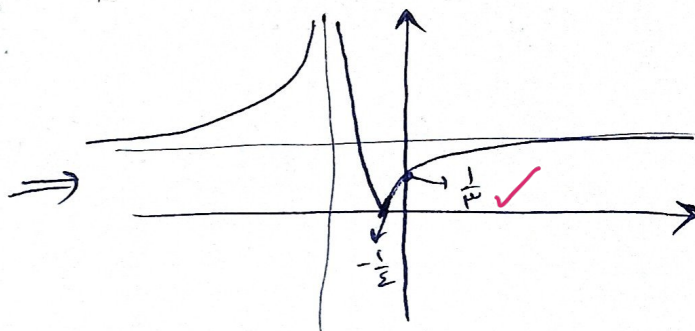
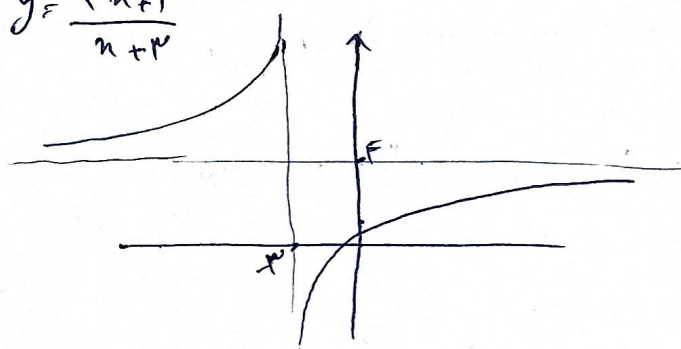
ب) $g(x) = 3 \cos^3 x - 4 = 3[-1, 1] - 4 = [-3, 3] - 4 = [-7, -1]$

$y = \frac{x}{|x|} + [x] \xrightarrow{x \geq 0} 1 + [x]$
 $\xrightarrow{x < 0} [x] - 1$



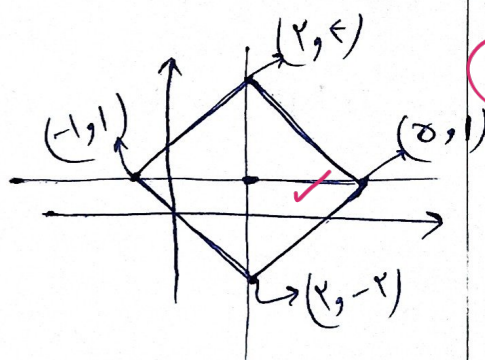
صفر تعریف شده است

$y = \frac{x+1}{x+2}$



$|x-2| + |y-1| = 3$

$\begin{cases} x \geq 2, y \geq 1 \rightarrow x-2+y-1=3 \rightarrow y = -x+6 \\ x \leq 2, y \geq 1 \rightarrow 2-x+y-1=3 \rightarrow y = x+2 \\ x \geq 2, y \leq 1 \rightarrow x-2+1-y=3 \rightarrow y = x-4 \\ x \leq 2, y \leq 1 \rightarrow 2-x+1-y=3 \rightarrow y = -x \end{cases}$



الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}} = (\sqrt{x + \frac{1}{x}} + 1)^2 - 1 \rightarrow (\sqrt{2} + 1)^2 - 1 = 2 + 2\sqrt{2}$
 $a + \frac{1}{a} \geq 2 \rightarrow (\sqrt{2} + 1)^2$

$R = [2 + 2\sqrt{2}, +\infty)$

ب) $y = -\frac{1}{x} + 2\sqrt{x + \frac{1}{x}} \rightarrow 2$

$\begin{matrix} -1 \rightarrow 0 \\ -\frac{b}{ra} = 1 \rightarrow 2 \end{matrix} \Rightarrow R = [0, 2]$

$$f(n) = -\frac{1}{n}n + 10 \quad D = \{1, 2, 3, \dots, 9\} \Rightarrow \text{ex.} = -\frac{1}{n}(1+2+3+\dots+9) + 9(10) = -\frac{1}{n}\left(\frac{9 \times 10}{2}\right) + 90 = -18 + 90 = 72$$

$$an^2 + bn + c \rightarrow b=0 \Rightarrow a=0 \Rightarrow bn+c=0 \rightarrow nb+c=0 \rightarrow nb=-c$$

$$n=3 \Rightarrow 3b+c=1 \rightarrow c=3b-1$$

$$g(n) = \sqrt{n^2 + 1} \rightarrow \text{ext}/\infty \Rightarrow R = \sqrt{[1, +\infty)} = [1, +\infty)$$

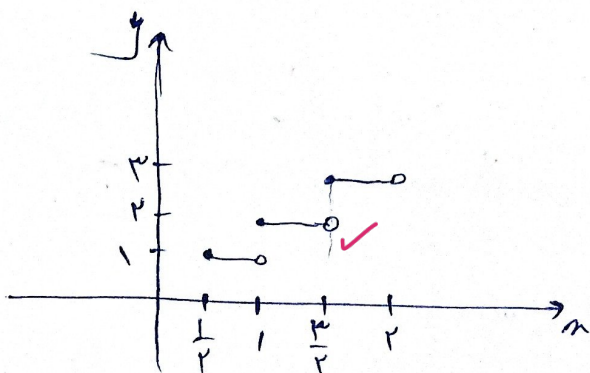
$$f(n) = \frac{n(n-1)(n+1)}{n+1} = n(n-1) \rightarrow n(n-1) = 1 \Rightarrow n=2$$

$$D_f = \mathbb{R} - \{-1, 1\} \quad R = n^2 - 2n \rightarrow \text{ext}/-1 \Rightarrow R = [-1, +\infty) - \{1\} \Rightarrow c = -1$$

$$f\left(\frac{a+b}{c}\right) = f\left(\frac{1}{-1}\right) = f\left(-\frac{1}{1}\right) = f(-1) = n(n-1) = -1(-2) = 2$$

$$\sqrt[3]{n^3 + an^2 + bn - 1} = \sqrt[3]{(n-1)^3} = n-1 \Rightarrow (n-1)^3 = n^3 - 3n^2 + 3n - 1$$

$$D = [-4, 11] \Rightarrow R = [-4-1, 11-1] = [-5, 10]$$



$b=0, c=3$