

الف) $f(x) = 2 \sin^2(x + \frac{\pi}{3}) - 1 \rightarrow -1 \leq \sin^2 \alpha \leq 1 \rightarrow 0 \leq \sin^2 \alpha \leq 1$
 $\rightarrow 0 \leq 2 \sin^2 \alpha - 1 \leq 1 \rightarrow [-1, 1]$

ب) $g(x) = 3 \cos^3 x - 4 \rightarrow -1 \leq \cos \alpha \leq 1 \rightarrow -1 \leq \cos^3 \alpha \leq 1 \rightarrow -3 \leq 3 \cos^3 \alpha - 4 \leq 3 - 4$
 $\rightarrow R_f = [-1, -4]$

$y = \frac{x}{|x|} + [x]$
 دامنه $[-3, 3]$
 جدول تغییرات:
 - $-3 \leq x < -2 \rightarrow -1 + (-3) = -4$
 - $-2 \leq x < -1 \rightarrow -1 + (-2) = -3$
 - $-1 \leq x < 0 \rightarrow -1 + (-1) = -2$
 - $0 \leq x < 1 \rightarrow +1 + (0) = 1$
 - $1 \leq x < 2 \rightarrow +1 + (1) = 2$
 - $2 \leq x < 3 \rightarrow +1 + (2) = 3$
 - $x = 3 \rightarrow 1 + 3 = 4$

$y = \left| \frac{x+1}{x+3} \right|$
 نمودار همگرا و افقی
 نمودار
 $x = -3$ (جنبه قائم)
 $y = 4$ (جنبه افقی)

$|x-2| + |y-1| = 3$
 ضلع مربع
 $\begin{cases} x \geq 2, y \geq 1 \rightarrow x-2+y-1=3 \rightarrow y=-x+6 \\ x \geq 2, y \leq 1 \rightarrow x-2-y+1=3 \rightarrow y=x-4 \\ x \leq 2, y \geq 1 \rightarrow -x+2+y-1=3 \rightarrow y=x+2 \\ x \leq 2, y \leq 1 \rightarrow -x+2-y+1=3 \rightarrow y=-x \end{cases}$

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{x}}$
 تغییر متغیر $x + \frac{1}{x} = t \rightarrow t' + \frac{1}{t} \rightarrow at' + b \ln t + c$
 $\Rightarrow t = \sqrt{x + \frac{1}{x}}$
 $\begin{cases} \min = 2 \rightarrow 2 + 2\sqrt{2} \\ \max = \infty \rightarrow \infty \end{cases} \rightarrow R_f = [2 + 2\sqrt{2}, \infty)$

ب) $y = (3 - \sin x)(1 + \sin x) \rightarrow 3 + 3\sin x - \sin x - \sin^2 x = -\sin^2 x + 2\sin x + 3$
 $\square = \sin x$
 $\begin{cases} \min = -1 \rightarrow -1 - 2 + 3 = 0 \\ \max = 1 \rightarrow -1 + 2 + 3 = 4 \end{cases} \rightarrow R_f = [0, 4]$

$f(x) = -\frac{1}{p}x + 10 \rightarrow$ نزول $\rightarrow$ تابع خطی

$D_f = \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \rightarrow (-\frac{1}{p}(1) + 10) + (-\frac{1}{p}(2) + 10) + \dots + (-\frac{1}{p}(9) + 10)$

مع: $R_f = ? \rightarrow R_f = -\frac{1}{p}(1+2+3+\dots+9) + (9 \times 10)$

$-\frac{45}{p} + 90 = 90 - 10 = 80$

$f(x) = \sqrt{ax^2 + bx + c}$ $\rightarrow$ $\begin{cases} a=0 \\ bx+c \end{cases}$ $\rightarrow$ $x=2$

$D_f = [2, +\infty)$

$f(2) = 1 \rightarrow \begin{cases} x=2 \rightarrow 0 \\ x=3 \rightarrow 1 \end{cases} \rightarrow \begin{cases} 2b+c=0 \\ 3b+c=1 \end{cases} \rightarrow \begin{cases} b=1 \\ c=-2 \end{cases}$

$g(x) = \sqrt{bx^2 + ax - c}$

$x=? \rightarrow \sqrt{x^2 - (-2)} = \sqrt{x^2 + 2} \Rightarrow R_f = [2, +\infty)$

$f(x) = \frac{x^3 - 5x}{x+2} \rightarrow \frac{x(x-2)(x+2)}{x+2} = x^2 - 2x$

$D_f = \mathbb{R} - \{a, b\} \rightarrow \begin{cases} a \\ b \end{cases} \rightarrow \begin{cases} a = -2 \\ b = 2 \end{cases}$

$R_f = [c, +\infty) - \{1\} \rightarrow a, b \text{ بین } c \text{ و } 1 \rightarrow x=b \rightarrow b^2 - 2b = 1 \rightarrow b = 2$

$f(\frac{a+b}{2}) \rightarrow c = y_s = 1 - \frac{b}{2} = 1 \rightarrow y_s = 1 - 2 = -1 \rightarrow c = -1$

$\rightarrow f(\frac{-2+2}{-2}) = f(\frac{2}{-2}) = f(-1) \rightarrow (-1)^2 - 2(-1) = 1 + 2 = 3$

$y = \sqrt[3]{x^3 + ax^2 + bx - 1} \rightarrow$ $\rightarrow$ $\rightarrow$ $\rightarrow$ $(x-2)^3 = x^3 - 6x^2 + 12x - 8$

$D_f = [a, b] \rightarrow \sqrt[3]{(x-2)^3} = x-2 \rightarrow \begin{cases} a = -4 \\ b = 12 \end{cases}$

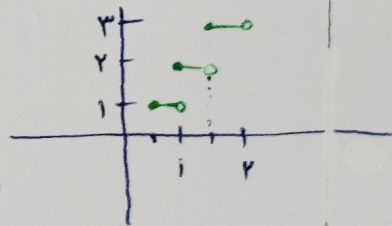
$R_f = ? \rightarrow D_f = [-4, 12] \rightarrow R_f = \begin{cases} x = -4 = -1 \\ x = 12 = 10 \end{cases} \rightarrow R_f = [-1, 10]$

$y = [x] + [x + \frac{1}{p}]$

$D = [\frac{1}{p}, 2) \rightarrow \begin{cases} \frac{1}{p} \leq x < 1 \rightarrow [x] + [x + \frac{1}{p}] = 1 \\ 1 \leq x < 1.5 \rightarrow [x] + [x + \frac{1}{p}] = 1 + 1 = 2 \end{cases}$

$\rightarrow$

$1.5 \leq x < 2 \rightarrow [x] + [x + \frac{1}{p}] = 1 + 2 = 3$



از 3 تا 2