

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{2}}$ $\xrightarrow{\sqrt{x + \frac{1}{2}} = t}$ $f(x) = t^2 + 2t + \frac{1}{t^2}$ $\xrightarrow{\text{min } t = \sqrt{x} \Rightarrow f(x) = 2 + 2\sqrt{x} - \infty}$
 $x > 0 \Rightarrow x + \frac{1}{2} \geq \sqrt{x} \Rightarrow x + \frac{1}{2} \geq \sqrt{x} \Rightarrow \sqrt{x + \frac{1}{2}} \geq \sqrt{x}$ $\xrightarrow{\text{max } t = +\infty \Rightarrow f(x) = +\infty}$
 $x < 0 \Rightarrow x + \frac{1}{2} \leq -\sqrt{x} \Rightarrow x + \frac{1}{2} \geq 0$ $\xrightarrow{t = -\frac{1}{\sqrt{x}} = -1 \text{ is not allowed}}$
 $\Rightarrow R_f = [1 + 2\sqrt{x}, +\infty)$

ب) $- \sin^2 x + 2 \sin x + 5$ $\begin{cases} \text{min}(\sin x) = -1 \Rightarrow y \geq 0 \\ \text{max}(\sin x) = 1 \Rightarrow y \geq 5 \\ -\frac{b}{2a} = 1 \Rightarrow y \geq 5 \end{cases} \Rightarrow R_f = [0, 5]$

(2) - 4

ج) $(1 + 2 + \dots + 9) = \frac{9 \times 10}{2} = 45$

$(-\frac{1}{3} \times 45) + (9 \times 10) = 90$ ✓

$R_f = [1, +\infty)$ $\xrightarrow{g = ax^2 + bx + c}$ $a \geq 0 \Rightarrow f(x) = \sqrt{bx + c} \Rightarrow f(x) = \sqrt{bx + c} \geq 1 \Rightarrow bx + c \geq 1$

(2) ✓

$f(x) = 0 = \sqrt{bx + c} \Rightarrow bx = -c \rightarrow b \leq 1, c \geq 1$

$\Rightarrow g(x) = \sqrt{x^2 + 5} \quad x^2 \geq 0 \Rightarrow x^2 + 5 \geq 5 \Rightarrow g(x) \geq \sqrt{5} \Rightarrow R_g = [\sqrt{5}, +\infty)$

$f(x) = \frac{(x+1)(x^2-2x)}{2+x} = \frac{x(x-1)(x+1)}{x+1} = x^2 - 2x; x \neq -1$

(2) ✓

$x^2 - 2x \neq 1 \Rightarrow x^2 - 2x - 1 \neq 0 \Rightarrow (x-1)(x+1) \neq 0 \Rightarrow x \neq -1, 1$

$R_f = \mathbb{R} - \{1, -1\} \Rightarrow a + b \geq 2$

$R_f = [-1, +\infty) - \{1\} \Rightarrow c \geq -1 \Rightarrow 1 \leq 2$

$f\left(\frac{a+b}{2}\right) = f\left(\frac{1}{2}\right) = f(-1)$

$f(-1) = 1 - 2(-1) = 3$

المثلثات المثلثية

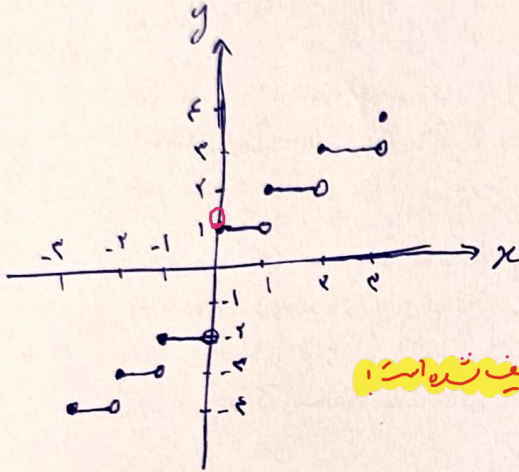
19, 5

$f(x) = 2 \sin(x + \frac{\pi}{4}) - 1$
 $\rightarrow \sin \pi = 0 \rightarrow f(x) = -1$
 $\rightarrow R_f = [-1, 1]$

(2) -1

$g(x) = 3 \cos x - 4$
 $\rightarrow \cos \pi = -1 \rightarrow g(x) = -7$
 $\rightarrow R_g = [-7, -1]$

$\rightarrow \cos 2\pi = 1 \rightarrow g(x) = -1$

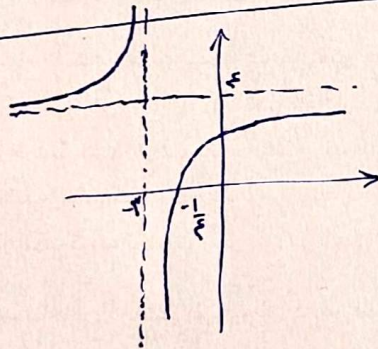


معرفة تعريف

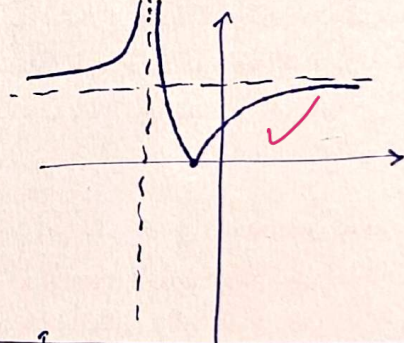
$$y = \begin{cases} -1-5 = -6 & -5 \leq x < -4 \\ -1-4 = -5 & -4 \leq x < -3 \\ -1-3 = -4 & -3 \leq x < -2 \\ -1-2 = -3 & -2 \leq x < -1 \\ -1-1 = -2 & -1 \leq x < 0 \\ -1-0 = -1 & 0 \leq x < 1 \\ -1-1 = -2 & 1 \leq x < 2 \\ -1-2 = -3 & 2 \leq x < 3 \\ -1-3 = -4 & 3 \leq x < 4 \\ -1-4 = -5 & 4 \leq x < 5 \\ -1-5 = -6 & x = 5 \end{cases}$$

(1, 1/5)

$y = \frac{5x+1}{x+5}$



$y = \left| \frac{5x+1}{x+5} \right|$



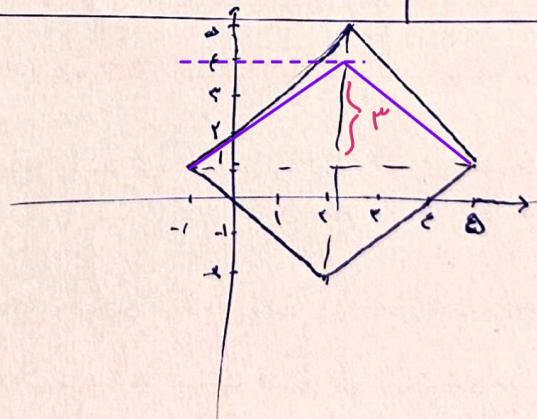
(2) 1/5

$2-4+y-1=5 \Rightarrow y = -2+6$

$-2+4+y-1=5 \Rightarrow y = x+4$

$-x+4-y+1=5 \Rightarrow y = -x$

$x-4+y+1=5 \Rightarrow y = 2-5$



(1, 1/5)