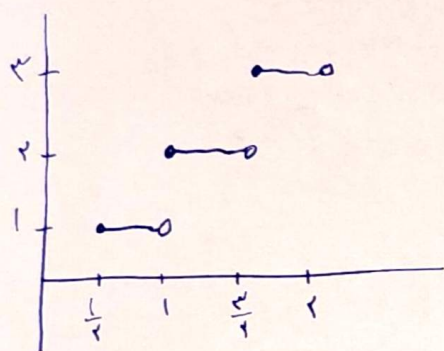


باقی به اینکه y خطی است پس تابع زیر را در یک شکل فرج به صورت $(x-a)^3$ است پس $a=2$ $\Rightarrow x=2$ $\Rightarrow x=2$ $\Rightarrow x=2$

$$x^3 + ax^2 + bx - 1 = (x-2)^3 = x^3 - 4x^2 + 12x - 8 \Rightarrow \begin{matrix} a = -4 \\ b = 12 \end{matrix} \Rightarrow y \in [-9, 12]$$

$$y = \sqrt[3]{(x-2)^3} = x-2 \Rightarrow \begin{matrix} x = -2 \rightarrow y = -4 \\ x = 12 \rightarrow y = 10 \end{matrix} \Rightarrow \text{Rf} \in [-4, 10]$$



$$y = \begin{cases} 0+1=1 & ; \frac{1}{4}(x < 1) \\ 1+1=2 & ; 1 \leq x < \frac{3}{4} \\ 1+2=3 & ; \frac{3}{4} \leq x < 1 \end{cases}$$

از ۳ پاره خط تشکیل شده است.

الف) $f(x) = x + \frac{1}{x} + 2\sqrt{x + \frac{1}{2}}$ $\xrightarrow{\sqrt{x + \frac{1}{2}} = t}$ $f(x) = t^2 + \frac{1}{t^2} + 2t$ $\xrightarrow{\text{min } t = \sqrt{2} \Rightarrow f(x) = 2 + 2\sqrt{2} = 2 + 2\sqrt{2}}$
 $x > 0 \Rightarrow x + \frac{1}{2} \geq \sqrt{2} \Rightarrow x + \frac{1}{2} \geq \sqrt{2} \Rightarrow \sqrt{x + \frac{1}{2}} \geq \sqrt{2} \xrightarrow{\text{max } t = +\infty \Rightarrow f(x) = +\infty}$
 $x < 0 \Rightarrow x + \frac{1}{2} \leq -\sqrt{2} \Rightarrow x + \frac{1}{2} \leq -\sqrt{2} \Rightarrow \sqrt{x + \frac{1}{2}} \leq -\sqrt{2} \xrightarrow{\text{min } t = -1 \Rightarrow f(x) = 2 + 2\sqrt{2}}$
 $\Rightarrow R_f = [2 + 2\sqrt{2}, +\infty)$

ب) $- \sin^2 x + 2 \sin x + 1$ $\begin{cases} \text{min}(\sin x) = -1 \Rightarrow y \geq 0 \\ \text{max}(\sin x) = 1 \Rightarrow y \leq 4 \\ -\frac{b}{2a} = 1 \Rightarrow y \geq 5 \end{cases} \Rightarrow R_f = [0, 4]$

-4

ج) $(1 + 2 + \dots + n) = \frac{n(n+1)}{2} = \epsilon \Rightarrow$

$(-\frac{1}{n} \times \epsilon) + (n \times 0) = 0 \Rightarrow \forall \epsilon$

$R_f = [1, +\infty)$ $\xrightarrow{g = ax^2 + bx + c}$ $a > 0 \Rightarrow f(x) = \sqrt{bx + c} \Rightarrow f(x) = \sqrt{bx + c} \geq 1 \Rightarrow bx + c \geq 1$

$f(x) = 0 = \sqrt{bx + c} \Rightarrow bx = -c \xrightarrow{b=1, c=-1}$

$\Rightarrow g(x) = \sqrt{x^2 - 1} \quad x^2 \geq 0 \Rightarrow x^2 - 1 \geq -1 \Rightarrow g(x) \geq 0 \Rightarrow R_g = [0, +\infty)$

$f(x) = \frac{(x+1)(x^2-1)}{x+1} = \frac{x(x-1)(x+1)}{x+1} = x^2 - x; x \neq -1$

$x^2 - x \neq 1 \Rightarrow x^2 - x - 1 \neq 0 \Rightarrow (x-1)(x+1) \neq 0 \Rightarrow x \neq -1, 1$

$R_f = \mathbb{R} - \{-1, 1\} \Rightarrow a+b=1$

$R_f = [-1, +\infty) - \{1\} \Rightarrow c=1 \Rightarrow 1c=1$

$f\left(\frac{a+b}{1c}\right) = f\left(\frac{1}{1}\right) = f(1)$

$f(-1) = 1 - 1(-1) = 2$

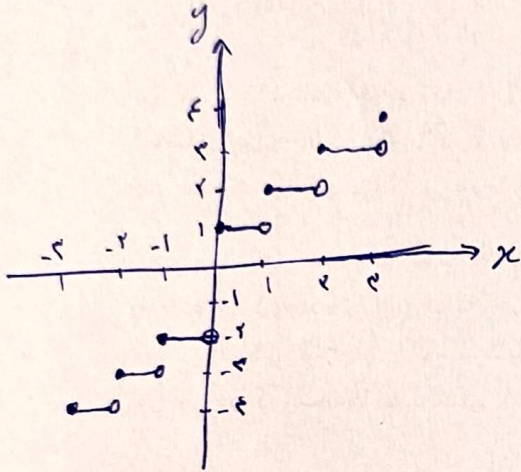
أوجد المجال

$$\text{حيث } f(x) = 4 \sin^2(x + \frac{\pi}{4}) - 1 \rightarrow \sin = 0 \rightarrow f(x) = -1 \Rightarrow R_f = [-1, 1]$$

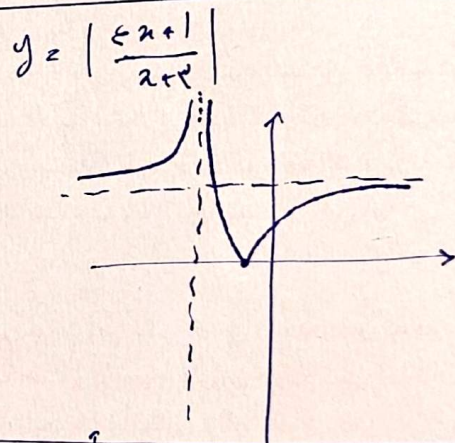
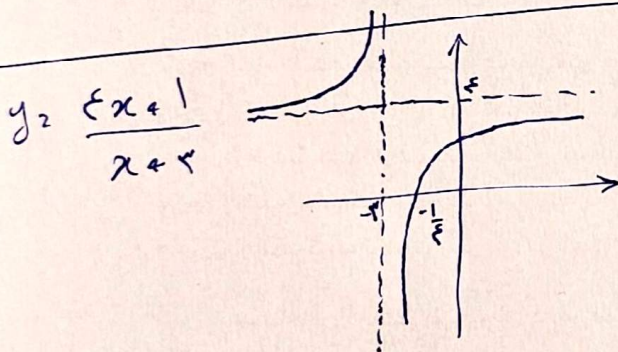
$$\rightarrow \sin = 1 \rightarrow f(x) = 1$$

$$\rightarrow g(x) = 4 \cos^2 x - 5 \rightarrow \cos = -1 \Rightarrow g(x) = -1$$

$$\rightarrow \cos = 1 \Rightarrow g(x) = -5 \Rightarrow R_g = [-5, -1]$$



$$y = \begin{cases} -1-5 = -6 & -5 \leq x < -4 \\ -1-4 = -5 & -4 \leq x < -3 \\ -1-1 = -2 & -1 \leq x < 0 \\ 1+0 = 1 & 0 \leq x < 1 \\ 1+1 = 2 & 1 \leq x < 2 \\ 1+4 = 5 & 2 \leq x < 3 \\ 1+5 = 6 & x = 3 \end{cases}$$



$$2-4 \leq y-1 \leq 5 \Rightarrow y = -2+4$$

$$-2+4+y-1 \leq 5 \Rightarrow y = x+4$$

$$-x+4-y+1 \leq 5 \Rightarrow y = -x$$

$$x-4 \leq y+1 \leq 5 \Rightarrow y = x-4$$

