

الف) $f(x) = 2\sin^2\left(x + \frac{\pi}{4}\right) - 1$

$0 \leq \sin^2 x \leq 1$

$0 \leq 2\sin^2 x \leq 2$

$-1 \leq 2\sin^2 x - 1 \leq 1$

$R_f = [-1, 1]$

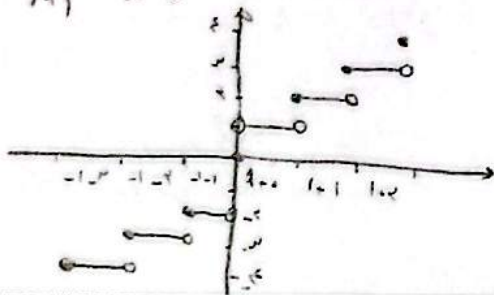
ب) $g(x) = 4\cos^3 x - 4$

$-1 \leq \cos^2 x \leq 1$

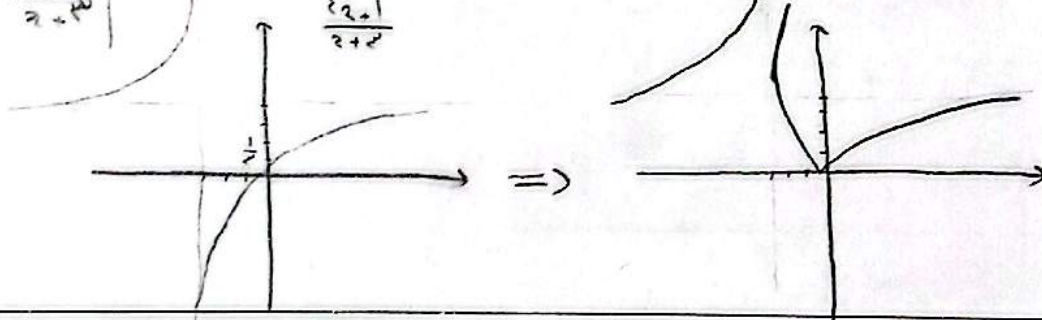
$-4 \leq 4\cos^2 x \leq 4$

$R_g = [-4, 0]$

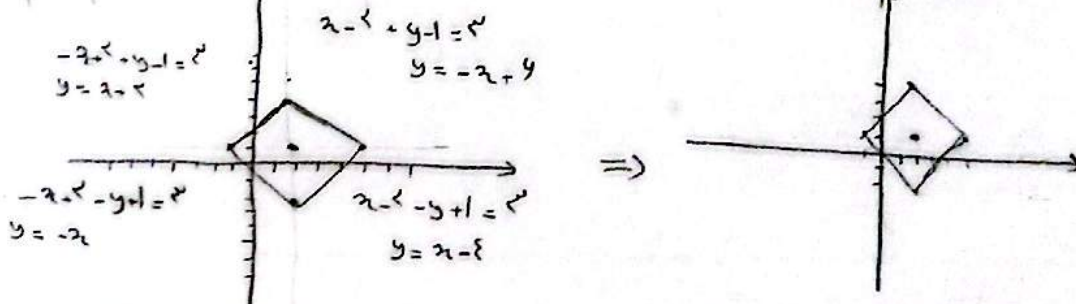
$y = \frac{x}{|x|} + [2]$



$y = \left| \frac{x+1}{x+2} \right|$



$|x-2| + |y-1| = 3$



الف) $x + \frac{1}{2} + \sqrt{x + \frac{1}{2}} \Rightarrow x + \frac{1}{2} \geq 0 \Rightarrow x + \frac{1}{2} \geq 1 \Rightarrow 1 + \sqrt{1} \Rightarrow R_f = [2, +\infty)$

ب) $(1 - \sin^2 x)(1 + \sin^2 x) = 1 - \sin^4 x = 1 - (\sin^2 x)^2 \Rightarrow R_f = [0, 1]$

$$f(x) = -\frac{1}{2}x + 1, \quad D_f = [1, 9]$$

$$f(1) = -\frac{1}{2}(1) + 1 = \frac{1}{2}, \quad f(4) = -\frac{1}{2}(4) + 1 = -1, \quad \dots, \quad f(9) = -\frac{1}{2}(9) + 1 = -\frac{7}{2}$$

$$\epsilon \times \frac{1}{2} = \frac{1}{2} \Rightarrow \frac{1}{2} = \frac{1}{2} = \boxed{\sqrt{0}}$$

$$f(x) = \sqrt{ax^2 + bx + c}$$

$$D_f = [9, \infty) \Rightarrow a(x-1)^2 = ax^2 - 2ax + a \quad f(4) = 9a - 12a + 4a = 1 \Rightarrow a = 1$$

$$\Rightarrow a = 1, \quad b = -2, \quad c = 1$$

$$D_f = \sqrt{bx^2 + ax - c} = \sqrt{-2x^2 + x - 1} \rightarrow \begin{cases} -\frac{2}{4a} = \frac{1}{4} \\ -\frac{1}{4a} = \frac{-12}{4} \end{cases} \Rightarrow \sqrt{(-\infty, -\frac{12}{1})} \Rightarrow R_f = \emptyset$$

$$f(x) = \frac{x^2 - 2x}{x + 2} = \frac{x(x-2)}{x+2} = x(x-2) \Rightarrow R_f = [-\frac{1}{2}, \infty) = [-1, \infty)$$

$$R_f = [-1, \infty) - \{1\} \Rightarrow \underline{c = -1}, \quad x(x-2) = 1$$

$$D_f = \mathbb{R} - \{a, b\} \Rightarrow \underline{a = -2}, \quad \underline{b = 1}$$

$$f\left(\frac{a+b}{2}\right) = \frac{-2+1}{-2} = \frac{-1}{-2} = \frac{1}{2} \Rightarrow \boxed{\frac{1}{2}} \text{ جواب نهایی}$$

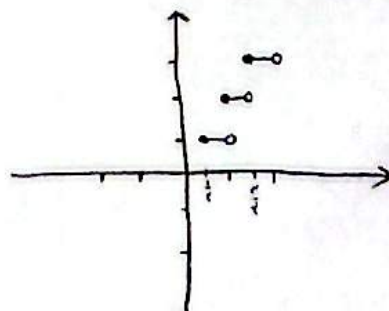
$$y = \sqrt{x^2 + ax^2 + bx - 1}$$

$$y = \sqrt{x(x-1)} \Rightarrow \sqrt{x^2 - 4x^2 + 12x - 1} \Rightarrow a = -4, \quad b = 12$$

$$D_f = [a, b] \Rightarrow \text{محدوده تعریف} \Rightarrow D_f = [-4, 12]$$

$$R_f = (-4, 12) = \boxed{[-1, 10]}$$

$$y = [x] + \left[x + \frac{1}{x}\right]$$



در 0, 1, 2, 3, 4