

۱- $0 < \sin^2(x + \frac{\pi}{3}) \leq 1 \rightarrow -1 < \sin(x + \frac{\pi}{3}) \leq 1$ (الف)

$\rightarrow 0 \leq 2 \sin^2(x + \frac{\pi}{3}) \leq 2 \rightarrow -1 \leq 2 \sin^2(x + \frac{\pi}{3}) - 1 \leq 1$

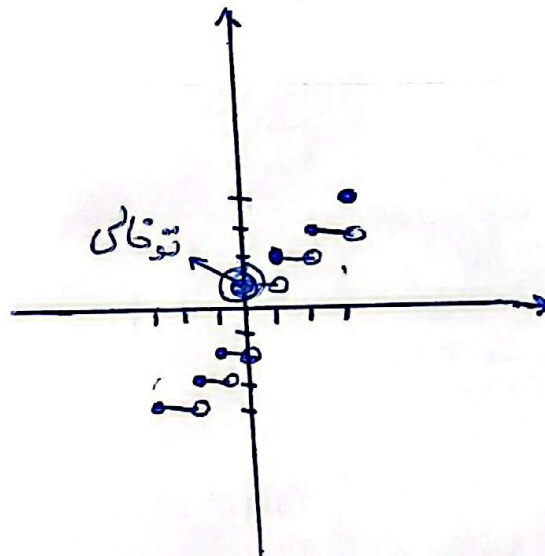
$\rightarrow R_f = [-1, 1]$

ب) $-1 < \cos x \leq 1 \rightarrow -1 \leq \cos^2 x \leq 1 \rightarrow -3 \leq 3 \cos^2 x \leq 3$

$\rightarrow -7 \leq 3 \cos^2 x - 4 \leq -1 \rightarrow R_f = [-7, -1]$

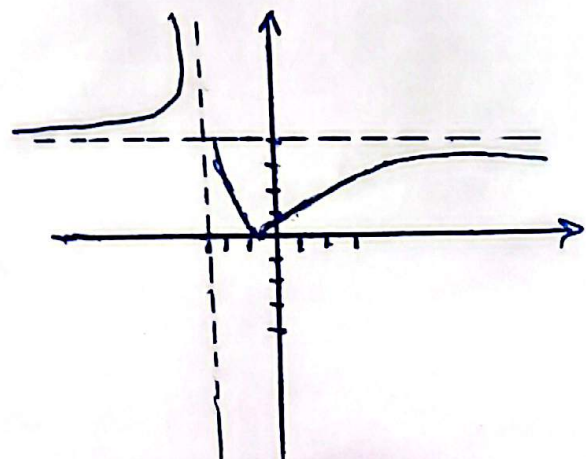
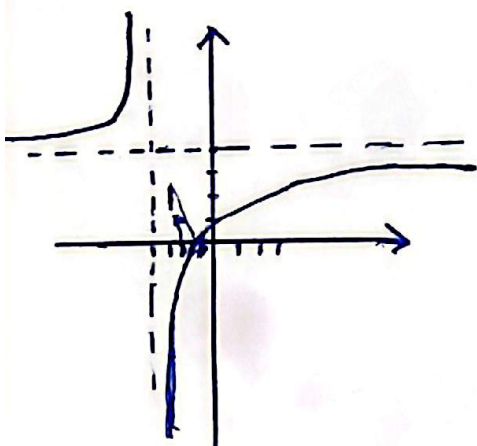
۲- $\frac{x}{|x|} \xrightarrow{-3 < x < 0} -1$ و $\frac{x}{|x|} \xrightarrow{0 < x < 3} 1$

$-3 < x < -2 \rightarrow -1 - 3 = -4$
 $-2 < x < -1 \rightarrow -1 - 2 = -3$
 $-1 < x < 0 \rightarrow -1 - 1 = -2$
 $0 < x < 1 \rightarrow 0 + 1 = 1$
 $1 < x < 2 \rightarrow 1 + 1 = 2$
 $2 < x < 3 \rightarrow 2 + 1 = 3$
 $x = 3 \rightarrow 3 + 1 = 4$

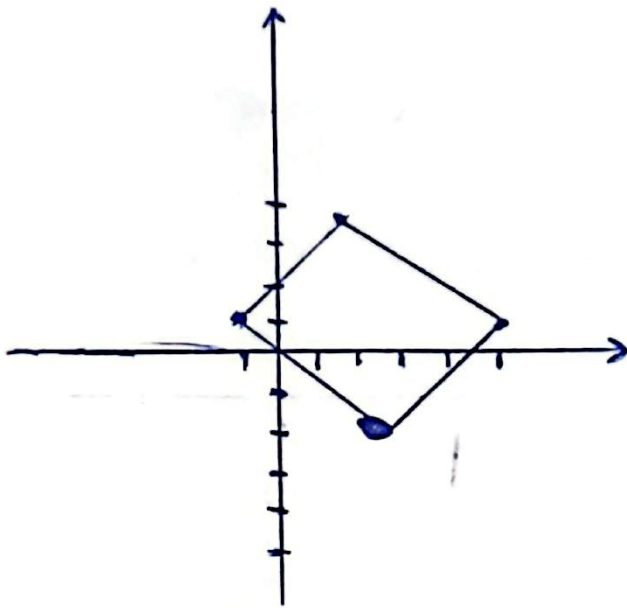


$y = \frac{4x+1}{x+3} \xrightarrow{x=0} \frac{1}{3} = y$

$y = \left| \frac{4x+1}{x+3} \right|$



$$|x-2|+|y-1|=3 \rightarrow y = \begin{cases} -x+4 & ; x \geq 2, y \geq 1 \\ x-4 & ; x \geq 2, y < 1 \\ x+2 & ; x < 2, y \geq 1 \\ -x & ; x < 2, y < 1 \end{cases}$$



$f(x) = x + \frac{1}{x} + \sqrt{x}$
 $\min \square = \sqrt{2}, \quad \square = \sqrt{\frac{x}{x}}$
 $\square = \sqrt{\frac{x}{x}} \rightarrow y = \square^2 + \sqrt{2}\square$
 $f(x) \xrightarrow[-\infty]{+\infty} \infty$
 $R_f = [1 + \sqrt{2}, +\infty)$

$y = (1 - \sin x)(1 + \sin x) \rightarrow y = -\sin^2 x + \sin x + \frac{1}{4}$
 $\sin x = -1$
 $\sin x = 1$
 $R_f = [0, \frac{5}{4}]$

$$y = -\frac{1}{4}x + 10$$

$x=1$	$-\frac{1}{4} + 10$
$x=2$	$-\frac{2}{4} + 10$
$x=3$	$-\frac{3}{4} + 10$

-4

$$\rightarrow \text{مجموع اعداد} = \underbrace{10+10+10+10+10+10+10+10+10}_{9(10)} - \frac{1}{4} \underbrace{(1+2+3+\dots+9)}_{45}$$

$$\rightarrow 90 - 15 = 75$$

✓ چون فقط ۲ به بعد تعریف می شود پس دارای یک مشتق است $\Leftarrow a=0$

$$6x + c \xrightarrow{x=2} 12b + c = 0 \rightarrow 12b = -c \rightarrow -12b = c$$

$$f(3) = \sqrt{12b + c} = 1 \rightarrow 12b + c = 1 \Rightarrow \boxed{b=1} \Rightarrow \boxed{c=-12b=-12}$$

$$g(x) = \sqrt{(1)x^2 + (0)x - 1(-12)}$$

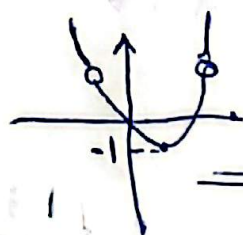
$$\rightarrow g(x) = \sqrt{x^2 + 12} \rightarrow \sqrt{x^2 + 12} \Rightarrow \boxed{D_g = [12, +\infty)}$$

$$f(x) = \frac{(x)(x-2)(x+2)}{x+2} \rightarrow f(x) = x^2 - 2x$$

$$\rightarrow x^2 - 2x \neq 1 \rightarrow x^2 - 2x - 1 \neq 0 \rightarrow (x-2)(x+2) \neq 0 \rightarrow \begin{cases} x \neq 2 \\ x \neq -2 \end{cases}$$

$$\rightarrow f(x) = x^2 - 2x$$

$$1 = -\frac{b}{2a} = x$$



$$\Rightarrow \boxed{c=-1}$$

$$\boxed{a=-2} \quad \boxed{b=2}$$

$$f\left(\frac{a+b}{2}\right) = f\left(\frac{-2+2}{2}\right) = f(0) = 0^2 - 2(0) = 0$$

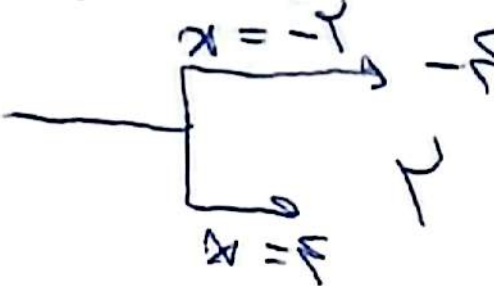
9 برای اینکه تابع خطی باشد باید به فرم $(x-2)^3$ باشد

$$\Rightarrow (x-2)^3 = x^3 + ax^2 + bx - 1$$

$$\rightarrow x^3 - 2x^2 + 4x - 1 = x^3 + ax^2 + bx - 1$$

$$a = -2 \quad b = 4$$

$y = x - 2$

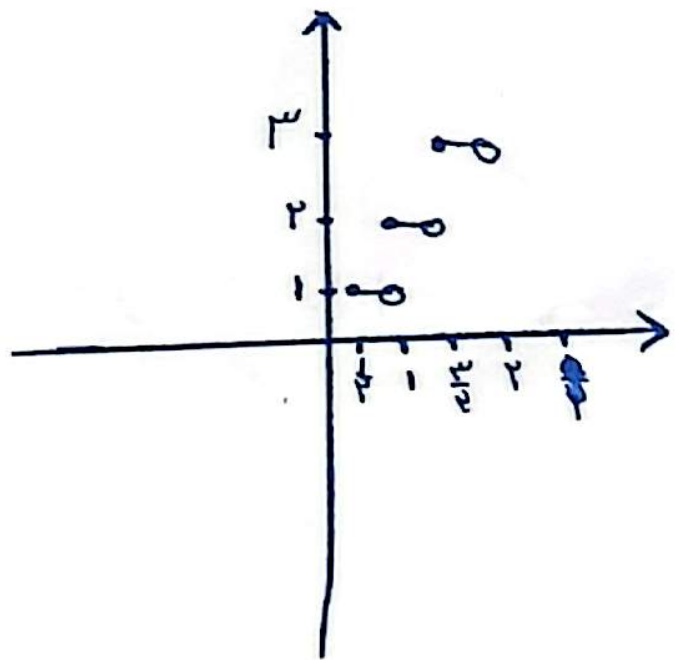


$$K_f = [-f \quad g \quad r]$$

-10

$$\frac{1}{2} \leq x < 1 \rightarrow 1 \leq x + \frac{1}{2} < \frac{3}{2} \rightarrow \frac{1}{2} \leq x < 1 \text{ and } 1 \leq x + \frac{1}{2} < \frac{3}{2}$$

$$1 \leq x < 2 \rightarrow 2 \leq x + \frac{1}{2} < \frac{5}{2} \rightarrow \frac{3}{2} \leq x < 2$$



$$y = \begin{cases} 1 & ; \frac{1}{2} \leq x < 1 \\ 2 & ; 1 \leq x < \frac{3}{2} \\ 3 & ; \frac{3}{2} \leq x < 2 \end{cases}$$