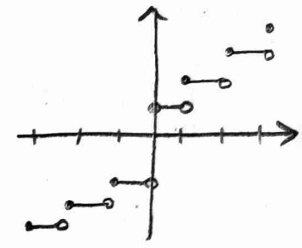
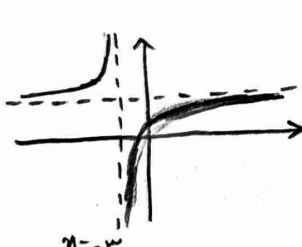
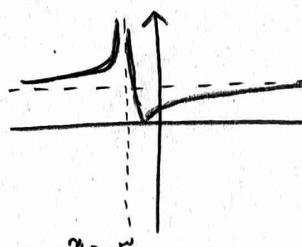
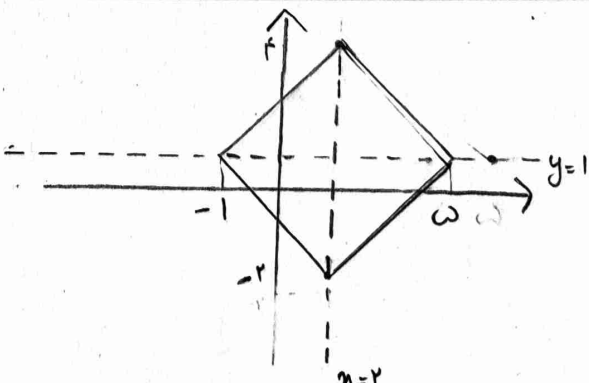


الف) $f(x) = 2 \sin^2(x + \frac{\pi}{3}) - 1$ $\sin^2 x \in [0, 1] \rightarrow 2 \sin^2 x \in [0, 2]$ $\rightarrow 2 \sin^2 x + 1 \in [1, 3]$	ب) $g(x) = 3 \cos^3 x - 4$ $\cos x \in [-1, 1] \rightarrow \cos^3 x \in [-1, 1]$ $3 \cos^3 x \in [-3, 3] \rightarrow 3 \cos^3 x - 4 \in [-7, -1]$	۱
$x \in [-3, -2) \rightarrow y = -1 - 3 = -4$ $x \in [-2, -1) \rightarrow y = -1 - 2 = -3$ $x \in [-1, 0) \rightarrow y = -1 - 1 = -2$ $x \in [0, 1) \rightarrow y = 1 + 0 = 1$	$x \in [1, 2) \rightarrow y = 1 + 1 = 2$ $x \in [2, 3) \rightarrow y = 1 + 2 = 3$ $x \in [3] \rightarrow y = 1 + 3 = 4$ 	۲
$y = \left \frac{x+1}{x+3} \right$ 		۳
$x < 2, y > 1 \rightarrow y = x + 2$ $x > 2, y > 1 \rightarrow y = 4 - x$ $x < 2, y < 1 \rightarrow y = -x$ $x > 2, y < 1 \rightarrow y = x - 2$		۴
الف) $f(x) = x + \frac{1}{x} + 2 \sqrt{x + \frac{1}{x}}$ $\sqrt{x + \frac{1}{x}} = t \rightarrow f(x) = t^2 + 2t$ $\max_t = \infty \rightarrow f(x) = \infty$ $\min_t = \sqrt{2} \rightarrow f(x) = 2 + 2\sqrt{2}$ $-\frac{1}{x} = t \rightarrow \infty$ $R_f \in [2 + 2\sqrt{2}, \infty)$	ب) $y = -\sin^2 x + 2 \sin x + 3$ $\max_{\sin x} = 1 \rightarrow y = 4$ $\min_{\sin x} = -1 \rightarrow y = 0$ $-\frac{1}{10} = \sin x \rightarrow y = 4$ $R_f \in [0, 4]$	۵

$$\{1, 2, \dots, 9\} = \text{اعداد طبیعی یک رقمی}$$

$$R_{f(961)} = -\frac{1+2+\dots+9}{9} + 9(10) = 90 - 15 = 75$$

۶

$$f(x) = \sqrt{ax^2 + bx + c}, D_f = [2, +\infty) \rightarrow \text{تابع است } f \rightarrow a=0, f(2)=0, f(3)=1$$

$$\rightarrow 2b+c=0, 3b+c=1 \rightarrow b=1, c=-2 \rightarrow g(x) = \sqrt{bx^2 - ax - 2c}$$

$$\rightarrow g(x) = \sqrt{x^2 + 2}, R_{x^2+2} \in [2, +\infty) \rightarrow R_g = \sqrt{[2, +\infty)} = [2, +\infty)$$

۷

$$f(x) = \frac{x^2 - 4x}{x+2} \rightarrow D_f \in \mathbb{R} - \{-2\} \rightarrow a, b = -2$$

$$\frac{x^2 - 4x}{x+2} = \frac{x(x+2)(x-2)}{x+2} = x^2 - 2x \rightarrow R_{x^2-2x} \in [-1, +\infty) \rightarrow R_f = R_{x^2-2x} - f(-2)$$

$$\rightarrow R_f = [-1, +\infty) - \{\underbrace{1}_c\} \rightarrow f\left(\frac{a+b}{2c}\right) = f(2) = 0$$

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$$a=-4, b=12 \leftarrow x^3 + ax^2 + bx - 8 = (x-2)^3 \leftarrow \text{تابع قطعی است}$$

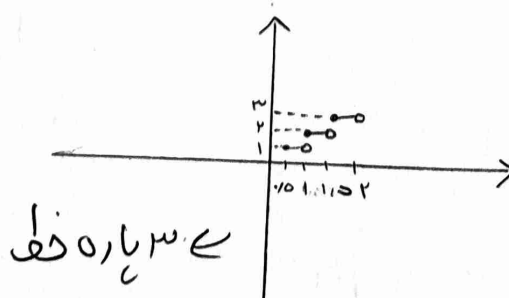
$$y = \sqrt[3]{(x-2)^3} = x-2, D_f = [-4, 12] \rightarrow R_f = [-8, 10]$$

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$$x \in [\frac{1}{2}, 1) \rightarrow y=1$$

$$x \in [1, 1.5) \rightarrow y=2$$

$$x \in [1.5, 2) \rightarrow y=3$$



$$0.5, 1.5 \in$$

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