

الف)  $\begin{cases} y_1^3 = 2x^2 + 2x - 1 \\ y_2^3 = 2x^2 + 2x - 1 \end{cases} \Rightarrow y_1^3 = y_2^3 \Rightarrow y_1 = y_2$  (تابع همت)

ب)  $\tilde{x} = \frac{11}{2} \rightarrow y = k\tilde{x} + \frac{11}{2} \rightarrow$  (تابع نیست)

ج)  $\begin{cases} x+y = 2\sqrt{xy} \rightarrow x+y-2\sqrt{xy} = 0 \\ \rightarrow (\sqrt{x}-\sqrt{y})^2 = 0 \rightarrow \sqrt{x}-\sqrt{y} = 0 \rightarrow \sqrt{x} = \sqrt{y} \\ \rightarrow x=y \end{cases}$  (تابع همت)

د)  $\begin{cases} (y^2+2)x - (y^2+2)y = (x-y)(y^2+2) = 0 \\ \div y^2+2 \rightarrow x-y=0 \rightarrow y=x \end{cases}$  (تابع همت)

$f(x) = \begin{cases} 2x^2 - b & x > 1 \\ 2x^2 - b & x \leq -1 \\ 2x + a & -3 \leq x \leq -1 \end{cases} \xrightarrow{x=-1} 2-b = -2+a \Rightarrow a+b=4$

با توجه به اینکه ضریب  $x^2$  منفی و  $a-bx-x^2 \geq 0$  و بر اساس جدول تعیین علامت آن  $a$  و  $b$  را پیدا می‌کنیم عبارت زیر را در یکال هستند:

$x=a \rightarrow a-ab-a^2=0$   
 $x=b \rightarrow a-2b^2=0 \Rightarrow a=2b^2$

فرض  $b \neq 0$   $\rightarrow 2b^2 - 2b^2 + 2b^2 = 2b^2(1-b+2b^2) = 0$   
 $\rightarrow b=0$  و  $a=0$

$\begin{cases} ax^2 + bx + c > 0 \\ D < 0 \end{cases} \Rightarrow \begin{cases} a=0 \\ 2b+c=0 \\ c=-2b \end{cases}$

$\xrightarrow{D < 0} \frac{0}{(-\infty, +\infty)} \rightarrow \frac{0}{+ \phi (-)}$

$\rightarrow a+c-3|b| = 2(0) + (-2b) - 3(-b) = -2+3b = |b|$  (چون  $b < 0$ )

$\rightarrow |x| - [x] \neq 0 \rightarrow |x| \neq [x]$

$\Rightarrow D_f = \mathbb{R} - \mathbb{Z}^+ - \{0\} = \mathbb{R} - (\mathbb{Z}^+ \cup \{0\})$

$\begin{cases} \rightarrow \sqrt{y} = 9 - \sqrt{x} \rightarrow y = x - 18\sqrt{x} + 81 \\ \rightarrow y = (9 - \sqrt{x})^2 \rightarrow D_f = [0, +\infty) = \mathbb{R}^+ \end{cases}$

$$\rightarrow \sqrt[3]{x-1} \geq 0 \rightarrow x-1 \geq 1 \rightarrow x \geq \frac{4}{3} \quad \left\{ \begin{array}{l} \rightarrow \frac{1}{9+\sqrt{|x|-|x|}} > 0 \rightarrow 9+\sqrt{|x|-|x|} > 0 \\ \sqrt{|x|}=t \rightarrow -t^2+t+9 > 0 \rightarrow t^2-t-9 < 0 \\ \rightarrow t \in (-2, 3) \text{ (منفی نمی تواند باشد)} \rightarrow t = \sqrt{|x|} \in [0, 3) \\ \rightarrow |x| \in [0, 9) \rightarrow x \in (-9, 9) \end{array} \right.$$

$$x > 0, x \neq 1 \Rightarrow D_f = \left( \frac{4}{3}, +\infty \right) - \{1\}$$

$$\sqrt[3]{x-1} > 0 \rightarrow x > \frac{4}{3}$$

$$\rightarrow \frac{\sqrt[3]{x+1} + a}{x+b} \geq 0 \rightarrow \frac{(a+\sqrt[3]{x+1})x + 1+ab}{x+b} \geq 0$$

$$D_f = \mathbb{R} - \left\{ \frac{d}{c} \right\} \quad x \neq -b \rightarrow b = -1$$

$$\rightarrow f(x) = \begin{cases} \sqrt[3]{x-1} & x \geq 0 \\ \sqrt[3]{x+1} & x < 0 \end{cases} \rightarrow \begin{cases} \sqrt[3]{x-1} \geq x-1 \rightarrow x \geq 0 \\ \sqrt[3]{x+1} \geq x-1 \rightarrow x \geq -\frac{4}{3} \end{cases}$$

$$f(x) - x + 1 \geq 0 \rightarrow f(x) \geq x-1$$

$$D_f = \left[ -\frac{4}{3}, +\infty \right) \leftarrow \left[ -\frac{4}{3}, 0 \right) \cup [0, +\infty)$$

$$y = \frac{\sqrt[3]{(\sqrt[3]{x+1} + \sqrt[3]{x+1})^3}}{(\sqrt[3]{x+1})^3} = \frac{\sqrt[3]{(x+1)^3}}{(\sqrt[3]{x+1})^3} = \frac{1}{\sqrt[3]{x+1}} \rightarrow \sqrt[3]{x+1} \neq 0$$

$$\rightarrow x \neq -\frac{1}{3} \rightarrow D = \mathbb{R} - \left\{ -\frac{1}{3} \right\}$$

$$\rightarrow \sqrt[3]{x-a} > 0 \rightarrow x > \frac{a}{3} \quad / \quad 1 - \sqrt[3]{x-a} \geq 0 \rightarrow 1 \geq \sqrt[3]{x-a}$$

$$\rightarrow 1 \geq \sqrt[3]{x-a} \rightarrow \frac{1+a}{3} \geq x$$

$$\Rightarrow \frac{a}{3} < x \leq 1 + \frac{a}{3} \rightarrow x \in \left( \frac{a}{3}, 1 + \frac{a}{3} \right]$$

$$(b, c] \Rightarrow c-b = 1 + \frac{a}{3} - \frac{a}{3} = 1$$