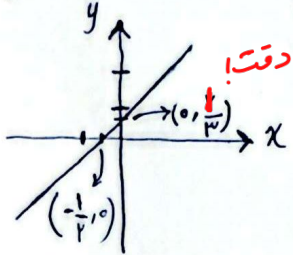


۱۹, ۲۵

نام و نام خانوادگی (شماره) پاسخنامه تشریحی تکلیف شماره کلاس دوازدهم پسر A

$$y = \frac{2}{3}x - 1 \rightarrow \frac{2y+1}{2} = x \rightarrow f^{-1}(x) = \frac{2}{3}x + \frac{1}{3}$$



با توجه به ضرایب وارون و نمودار رسم شده

$$S = \frac{\frac{1}{3} \times \frac{1}{2}}{2} = \frac{1}{12}$$

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الف) $x_s = -\frac{b}{2a} = -\frac{-2}{2 \times 1} = 1 \rightarrow y_s = 2 \rightarrow S(1, 2)$ (ب) با توجه به توصیفات (الف) یک به یک است

دامنه از $x \in \mathbb{R}$ به $y \in \mathbb{R}$ نظر گرفته شده پس یک به یک است

$D_f = [1, +\infty)$ $R_f = [2, +\infty) \rightarrow f(x) = (x-1)^2 + 2$

$y = x^2 - 2x + 3 = (x-1)^2 + 2$

نی توان از رادیکال منفی باشد

$\rightarrow f^{-1}(x) = \sqrt{x-2} + 1$ (چون هم تابع و وارونش صعودی است)

$\rightarrow f^{-1}(x) = \sqrt{x-2} + 1$

$\rightarrow D_{f^{-1}} = R_f = [2, +\infty)$

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$2x - \sqrt{14 - 4x(4-x)} = 2x - 2\sqrt{4 - 4x + x^2} = 2x - 2|x-2|$

$\begin{cases} x \geq 2 \\ x < 2 \end{cases}$

$f(x) = 4x - 4, x \in (-\infty, 2] = D_f \rightarrow R_f = (-\infty, 4]$

$\rightarrow f^{-1}(x) = \frac{1}{4}x + 1, x \in (-\infty, 4]$

$S = \frac{(1+2) \times 4}{2} = 6$

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$f(x_1) = f(x_2) \Rightarrow \frac{x_1}{\sqrt{x_1^2 - 5}} = \frac{x_2}{\sqrt{x_2^2 - 5}} \xrightarrow{x_1^2 = x_2^2} \frac{x_1^2 - 5}{x_1^2 - 5} = \frac{x_2^2 - 5}{x_2^2 - 5}$

$\rightarrow x_1^2 x_2^2 - 5x_1^2 = x_1^2 x_2^2 - 5x_2^2 \rightarrow x_1^2 = x_2^2 \rightarrow x_1 = \pm x_2$

$\rightarrow f(x) = y = \frac{x}{\sqrt{x^2 - 5}} \xrightarrow{y^2 = \frac{x^2}{x^2 - 5}} y^2 = \frac{x^2}{x^2 - 5} \rightarrow x^2 = \frac{5y^2}{y^2 - 1} \xrightarrow{\text{منفی نیست}} f^{-1}(x) = \sqrt{\frac{5x^2}{x^2 - 1}}$

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$g(x) = \begin{cases} x^2 - 4x + k & x \geq 2 \rightarrow x_{\min} = 2 \rightarrow y_{\min} = k - 4 \rightarrow R = [k - 4, +\infty) \\ -x^2 + 4x + 3k & x < 2 \rightarrow x = 2 \rightarrow y = 3k + 4 \rightarrow R = (-\infty, 3k + 4) \end{cases}$

$\Rightarrow 3k + 4 \leq k - 4 \rightarrow 2k \leq -8 \rightarrow k \leq -4 \rightarrow k \in (-\infty, -4]$

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$$f(x) = \begin{cases} 4x & x > 0, R = [0, +\infty) \\ 2x & x \leq 0, R = (-\infty, 0] \end{cases} \Rightarrow f^{-1}(x) = g(x) = \begin{cases} \frac{1}{4}x & x > 0 \\ \frac{1}{2}x & x \leq 0 \end{cases}$$

$$\Rightarrow g(x) = \boxed{\frac{3}{1}}x - \left(\frac{1}{1}\right)|x| = \boxed{a}x + \boxed{b}|x| \Rightarrow ab = \left(\frac{3}{1}\right) \cdot \left(-\frac{1}{1}\right) = -\frac{3}{1}$$

$$f(x) = \frac{3x+2}{x+1} \xrightarrow{(1,1)} \frac{1}{2} = -1 \rightarrow m = -3 \rightarrow f(x) = \frac{3x+2}{x-3}$$

$$y = f \circ f^{-1}(x) + f^{-1}(x) = \left\{ x + \frac{3x+2}{x-3} \right\} \Rightarrow x + \frac{3x+2}{x-3} = x$$

$$\rightarrow \frac{3x+2}{x-3} = 0 \rightarrow 3x+2=0 \rightarrow x = -\frac{2}{3}$$

$$\rightarrow g\left(\frac{x}{2}\right) = t \rightarrow f(3-2x) = 2-t \xrightarrow{f^{-1}} 3-2x = f^{-1}(2-t) \rightarrow \frac{x}{2} = \frac{3-f^{-1}(2-t)}{2}$$

$$= g^{-1}(t) \rightarrow 2g^{-1}(t) = 3 - f^{-1}(2-t) \xrightarrow{t=2} 2g^{-1}(2) = 3 - f^{-1}(0)$$

$$2g^{-1}(2) + f^{-1}(0) = 3 - f^{-1}(0) + f^{-1}(0) = 3$$

$$y = \frac{2x-13}{1-x} \xrightarrow{\text{قرینه نسبتی}} -y = \frac{-2x+13}{x+1} = \frac{2x+13}{x+1} \xrightarrow{\text{مضرب 2}} \frac{2(x-1)+15}{x-1} = \frac{2x+13}{x-1}$$

$$\rightarrow \frac{2x-13}{x-1} - 13 = \frac{2x}{x-1} = f \rightarrow f^{-1}(x) = \frac{x}{x-2} \Rightarrow \frac{2x}{x-1} = \frac{x}{x-2}$$

$$\rightarrow x^2 - x = 2x^2 - 4x \rightarrow x^2 - 3x = 0 \rightarrow S = -\frac{b}{a} = -\frac{-3}{1} = 3$$

$$\rightarrow y = x + f[x] \rightarrow [y] = [x + f[x]] = 2[x] \rightarrow [x] = \frac{[y]}{2} \leftarrow f^{-1}(x) = g(x)$$

$$\hookrightarrow y = x + f\left(\frac{[y]}{2}\right) \rightarrow x = y - 0.1[y] \rightarrow f^{-1}(x) = x - 0.1[x]$$

$$(f-g)(x) = x + f[x] - x - 0.1[x] = 0.9[x]$$